Notes on moving mirrors

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The Davies-Fulling (DF) model describes the scattering of a massless field by a noninertial mirror in two dimensions. In this paper, we generalize this model in two different ways. First, we consider partially reflecting mirrors. We show that the Bogoliubov coefficients relating inertial modes can be expressed in terms of the reflection factor and the transformation from inertial modes to modes at rest with respect to the mirror. In this perspective, the DF model is simply the limiting case when the reflection factor is unity for all frequencies. In the second part, we introduce an alternative model which is based on self-interactions described by an action principle. When the coupling is constant, this model can be solved exactly and gives rise to a partially reflecting mirror. The usefulness of this dynamical model lies in the possibility of switching off the coupling between the mirror and field. This allows us to obtain regularized expressions for the fluxes in situations where they are singular when using the DF model. Two examples are considered. The first concerns the flux induced by the disappearance of the reflection condition, a situation which bears some analogies with the end of the evaporation of a black hole. The second case concerns the flux emitted by a uniformly accelerated mirror.

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I. INTRODUCTION

The Davies-Fulling (DF) model [1] describes the scattering of a massless field by a noninertial mirror in two dimensions. It has received and continues to receive attention [2–11] principally because of its simplicity and its relationship to Hawking radiation [12]. (One can indeed mimic the varying Doppler effect induced by the collapse of a star by the total reflection on a receding mirror.) Because of its simplicity, this model has been also used to investigate the relationships between the particle description of fluxes emitted by the mirror and its field description based on the two-point Green's function. The motivation behind this analysis is the following. When quantizing a field in a curved space-time, one loses the uniqueness of choice for the particle notion which is then used to define the vacuum and its excitations. Based on this fact, some authors have proposed discarding the particle point of view [13]. The DF model, being defined in flat space time and giving rise to particle creation as in a curved space-time, provides a good playground for confronting the two points of view. Finally, the DF model also provides a good starting point for studying the role of ultrahigh frequencies which arise in the presence of event horizons [14–18]. This is particularly true when considering uniformly accelerated mirrors [3,19,20]. Indeed, in this case one has to confront the fact that the instantaneous value of the energy flux identically vanishes, whereas the Bogoliubov coefficients, mixing positive and negative frequencies, do not vanish and lead to a total energy, which furthermore diverges.

Quite independently of these specific difficulties, there is a fundamental reason which complicates the analysis of these problems: the DF model does not follow from an action principle. In fact, the reflection condition is imposed from the outset instead of following from interactions with the boundary. Therefore only questions concerning asymptotic properties of asymptotically inertial mirrors can be properly answered. To emphasize this point, we shall show in the first part of this article that the scattering in the DF model can be expressed in purely kinematic terms. It results from the Bogoliubov transformation relating the usual Minkowski modes to noninertial modes which are eigenmodes with respect to the proper time of the mirror. The scattering of the latter is then trivial, as trivial as the scattering of Minkowski modes by an inertial mirror. This rephrasing of the DF model is very useful in that it allows us to consider partially transmitting mirrors with arbitrary frequency-dependent transmission coefficients. In this perspective, the DF model is simply the limiting case in which the reflection is total for all frequencies.

In the second part of the paper, we analyze an alternative model for scattering along a given trajectory which is based on self-interactions described by an action principle. The main motivation for considering this model is that we can now switch on and off the coupling between the mirror and field. Therefore, we can work with well-defined asymptotic free states. The relationship between the partially transmitting mirrors previously considered and this model will be explicitly made.

To this end, we first work with a coupling which is constant. In this case, the Born series can be exactly summed and lead to a partially transmitting mirror. Moreover, in the large coupling constant limit, one recovers the DF model: i.e., total reflection. The only difference with respect to the kinematic approach is that causality is now built in. Second, we consider the case when the coupling is time dependent. In this case, we compute the fluxes perturbatively to quadratic order in the coupling. The novelty arises from transient effects associated with the switching on and off. The possibility of controlling these transients is crucial for regularizing the fluxes in situations where they are singular when using the DF model.

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To make this explicit, we consider two examples. The first one consists in computing the flux associated with the appearance (or disappearance) of the reflecting boundary condition. This problem was considered by Anderson and DeWitt [21]. Moreover, as discussed in [7], it presents some analogies with the residual flux associated with the disappearance of a black hole at the end of the evaporation process. When using the DF model, the flux is singular and its spectral properties are ill defined. On the contrary, with the self-interacting model, it can be described by a well-defined process in which the coupling to the mirror is switched off more and more rapidly. The second application concerns the flux emitted by a uniformly accelerated mirror. In the DF model, the energy flux vanishes everywhere, but on the horizons where it is not defined. With the other model, instead, a well-defined and regular expression will be obtained. In the intermediate regime, when the coupling is constant, we recover the vanishing of the local flux. But we also find transient effects which become singular when the switching on and off is performed for asymptotic early and late proper times, thereby explaining the paradoxical situation encountered in the DF model where quanta are produced while the energy flux vanishes.

We conclude the paper by presenting the main results in a synthetic manner. We also wish to stress that in this paper recoil effects shall be totally ignored since the trajectory of the mirror is given once for all. Nevertheless, since the self-interacting model is based on Feynman diagrams, it prepares for the analysis of taking into account the dynamics of the mirror [10,11]. Indeed, the S matrices computed with or without backreaction effects possess a very similar structure.

II. KINEMATIC MODELS

In the first part of this section, we review the basic properties of the Davies-Fulling model. In particular, we compare the particle description of the fluxes based on Bogoliubov coefficients with that based on two-point functions. In the second part, we show how the scattering process can be generalized so as to describe partially transmitting mirrors. This generalization will be performed in a matrix formalism. We have chosen this formalism for two reasons: first to emphasize the kinematic nature of the DF model and second to introduce in natural terms the generalization to partial reflection. In the third part, we relate the Bogoliubov coefficients to the S matrix acting in Fock space, thereby preparing for the analysis of transition amplitudes performed in the next section.

A. Davies-Fulling model

In the Davies-Fulling model, the mirror is perfectly reflecting for all frequencies and its trajectory is chosen from the outset. Moreover, no width is attributed to the reflecting condition: i.e., it acts like a delta in space. Beside the fact that the trajectory is always timelike, we shall also impose that it be asymptotically inertial. In conformal terms this means that the trajectory starts from i^- and ends in i^+, the past and future timelike infinities, respectively [22]. The reason is that in the other cases, i.e., when the mirror originates and/or ends on null infinities, the calculation of the energy radiated by the mirror is ill defined. (The specific problems associated with null asymptotic trajectories will be considered in a next article [23].)

In this paper, we shall consider the scattering of a *complex* massless scaler field. The reason for this choice is that it allows us to consider more general scattering matrices when the reflection condition is not perfect. This possibility will be exploited in the next subsections. Since the dynamics of the mirror is trivial, the evolution of the field is entirely governed by the d'Alembert equation

$$(\partial_t^2-\partial_z^2)\Phi(t,z)=0 \tag{1}$$

and the reflection condition

$$\Phi(t,z_{\rm cl}(t))=0 \tag{2}$$

along the classical trajectory of the mirror $z=z_{\rm cl}(t)$.

Since the field is massless and since we work in two dimensions, it is particularly useful to work in the lightlike coordinates defined by U, $V=t\mp z$. For instance, the general solution of Eq. (1) is the sum of a function of U alone plus a function of V. In addition, since the mirror is perfectly reflecting, the trajectory of the mirror completely decouples the left-hand-side configurations from the right-hand-side ones. Therefore, in this subsection, we can and shall restrict our attention to the configurations residing on the left of the mirror.

Finally, since the mirror trajectory emerges from i^-, $V=-\infty$ is a complete Cauchy surface. Hence the usual modes of the d'Alembertian given by

$$\varphi_k(U)=\frac{e^{-ikU}}{\sqrt{4\pi|k|}} \tag{3}$$

form a complete and orthonormal basis. (Instead, when the trajectory starts from the null past infinity $\mathcal{J}^-$, the choice of a complete and orthonormal basis should be reconsidered [23].) We recall that the norm of the modes is determined by the Klein-Gordon scalar product which reads, when evaluated on $\mathcal{J}^-$,

$$\langle\varphi_k|\varphi_{k'}\rangle=\int_{-\infty}^{+\infty}dU\,\varphi_k^* i\overleftrightarrow{\partial}_U\varphi_{k'}={\rm sgn}(k)\,\delta(k-k'). \tag{4}$$

The scattered mode corresponding to Eq. (3) is determined by Eq. (2) to be

$$\varphi_k^{\rm scat}(V)=-\frac{e^{-ikU_{\rm cl}(V)}}{\sqrt{4\pi|k|}}, \tag{5}$$

where $U=U_{\rm cl}(V)$ is the trajectory of the mirror in the lightlike coordinates.

The in mode $\varphi_k^{\rm in}(U,V)$ is by definition the solution of Eqs. (1) and (2) which has Eq. (3) as initial data. It is given by

$$\varphi_k^{\rm in}(U,V)=\frac{e^{-ikU}}{\sqrt{4\pi|k|}}-\frac{e^{-ikU_{\rm cl}(V)}}{\sqrt{4\pi|k|}}. \tag{6}$$

To analyze the frequency content of its scattered part, it should be Fourier decomposed on the final Cauchy surface $U=+\infty$ (the left part of $\mathcal{J}^+$). In total analogy with what we have on $\mathcal{J}^-$, on $\mathcal{J}^+$ the normalized modes are

$$\varphi_\omega(V)=\frac{e^{-i\omega V}}{\sqrt{4\pi|\omega|}}. \tag{7}$$

Then the scattered mode (5) can be decomposed as

$$\varphi_k^{\rm scat}=\int_0^\infty d\omega(\alpha_{\omega k}^*\varphi_\omega-\beta_{\omega k}^*\varphi_\omega^*), \tag{8}$$

where the coefficients $\alpha_{\omega k}$, $\beta_{\omega k}$ are given by the overlaps

$$\alpha_{\omega k}^*=\langle\varphi_\omega|\varphi_k^{\rm scat}\rangle=-2\int_{-\infty}^{+\infty}dV\frac{e^{i\omega V}}{\sqrt{4\pi|\omega|^{-1}}}\frac{e^{-ikU_{\rm cl}(V)}}{\sqrt{4\pi|k|}},$$

$$\beta_{\omega k}^*=\langle\varphi_\omega^*|\varphi_k^{\rm scat}\rangle. \tag{9}$$

Since both the initial and final sets of modes are complete, the coefficients $\alpha_{\omega k}$, $\beta_{\omega k}$ satisfy the relations

$$\int_0^\infty dk(\alpha_{\omega k}^*\alpha_{\omega' k}-\beta_{\omega k}\beta_{\omega' k}^*)=\delta(\omega-\omega'),$$

$$\int_0^\infty d\omega(\alpha_{\omega k}\alpha_{\omega k'}^*-\beta_{\omega k}\beta_{\omega k'}^*)=\delta(k-k'),$$

$$\int_0^\infty dk(\alpha_{\omega k}\beta_{\omega k'}-\beta_{\omega k}\alpha_{\omega k'})=0,$$

$$\int_0^\infty d\omega(\alpha_{\omega k}\beta_{\omega' k}^*-\beta_{\omega k}^*\alpha_{\omega' k})=0. \tag{10}$$

Note that these relations are not trivially satisfied when the trajectory of the mirror reaches one of the null infinities rather than the timelike ones. Notice also that the overlaps (9) can be computed on any spacelike surface which runs from $z=-\infty$ to some point on the mirror $(t,z_{\rm cl}(t))$. In this case, one should use the full expression of the in modes given in Eq. (6) as well as that of the out modes given by

$$\varphi_\omega^{\rm out}(U,V)=\varphi_\omega(V)+\varphi_\omega^{b{\rm scat}}(U). \tag{11}$$

The second term $\varphi_\omega^{b{\rm scat}}$ results from the backward scattering of φ_ω given in Eq. (7).

When the overlaps $\alpha_{\omega k}$ and $\beta_{\omega k}$ are known, the classical scattering problem is solved. That is, it suffices to decompose the initial data in terms of the modes (7) to obtain, through Eq. (9), the Fourier content of its image on $\mathcal{J}^+$. It should be pointed out that the coefficients $\beta_{\omega k}$ which mix positive and negative frequencies have a well-defined role in this classical wave theory: they determine the (nonadiabatic [24]) increase of the Fourier components of the scattered wave [see, e.g., Eq. (11) in [10] for their influence on the energy of the reflected wave]. It should be also pointed out that one can recover an approximate space-time description of the scattering when considering localized wave packets rather than plane waves: for sufficiently high frequencies [i.e., short wavelengths compared to the (acceleration of the mirror)$^{-1}$], the coefficients $\beta_{\omega k}$ vanish and the mean frequency of the reflected packet $\bar{\omega}$ is related to $\bar{k}$, that of the incident one, by the Doppler effect $\bar{\omega}=\bar{k}\partial_V U_{\rm cl}|_{U=\bar{U}}$ evaluated at $\bar{U}$, the retarded time of the center of the incident packet. These two properties are easily obtained by forming wave packets and evaluating the integrals in Eq. (9) by the saddle point method.

When $\alpha_{\omega k}$ and $\beta_{\omega k}$ are known, the quantum scattering problem is also solved. This follows from the linearity of Eqs. (1) and (2): when working in a second-quantized framework, being linear, these equations provide the Heisenberg equations for the field operator. Thus the field operator can be written both in the in and out bases by

$$\Phi=\int_0^\infty dk(a_k^{\rm in}\varphi_k^{\rm in}+b_k^{\rm in\dagger}+\varphi_k^{\rm in}*)$$

$$=\int_0^\infty d\omega(a_\omega^{\rm out}\varphi_\omega^{\rm out}+b_\omega^{\rm out\dagger}\varphi_\omega^{\rm out}*). \tag{12}$$

When imposing that it satisfy the equal-time commutation relation $[\Phi(z),\partial_t\Phi^\dagger(z')]=i\delta(z-z')$, Eq. (4) guarantees that the in operators a_k, b_k satisfy the usual commutation relations leading to the particle interpretation. Then the in vacuum $|0_{\rm in}\rangle$ is defined as the product of the ground states of the in oscillators and its excitations are generated by the creation operators $a_k^{\rm in\dagger}$, $b_k^{\rm in\dagger}$. Moreover, by construction of the in modes on $\mathcal{J}^-$, the in particles correspond to the usual Minkowski particles on $\mathcal{J}^-$. Similarly, by construction of the out modes, all these properties apply to the out operators $a_k^{\rm out\dagger}$, $b_k^{\rm out\dagger}$ and to the out vacuum $|0_{\rm out}\rangle$ when replacing $\mathcal{J}^-$ by $\mathcal{J}^+$.

Given the orthonormal and complete character of the in and out mode basis, Eqs. (9) and (12) determine the Bogoliubov relations

$$a_k^{\rm in}=\int_0^\infty d\omega(\alpha_{\omega k}a_\omega^{\rm out}+\beta_{\omega k}b_\omega^{\rm out\dagger}),$$

$$b_k^{\rm in\dagger}=\int_0^\infty d\omega(\beta_{\omega k}^*a_\omega^{\rm out}+\alpha_{\omega k}^*b_\omega^{\rm out\dagger}),$$

$$a_\omega^{\rm out}=\int_0^\infty dk(\alpha_{\omega k}^*a_k^{\rm in}-\beta_{\omega k}b_k^{\rm in\dagger}),$$

$$b_\omega^{\rm out\dagger}=\int_0^\infty dk(-\beta_{\omega k}^*a_k^{\rm in}+\alpha_{\omega k}b_k^{\rm in\dagger}). \tag{13}$$

Then Eqs. (10) guarantee the compatibility of the particle interpretation in each basis, i.e., both in and out operators obey the canonical commutations relations. With the relations (13), all questions concerning quantum scattering processes can be answered. For instance, the probability amplitude to obtain a given final state $|\Psi_{\rm fin}\rangle$ specified on $\mathcal{J}^+$ in terms of out operators starting from some in state $|\Xi_{\rm in}\rangle$ constructed on $\mathcal{J}^-$ is given by the product $\langle\Psi_{\rm fin}|\Xi_{\rm in}\rangle$ [which should not be confused with the Klein-Gordon product, Eq. (4), which concerns the modes of the field]. More intrinsic is the overlap $Z^{-1}=\langle 0_{\rm out}|0_{\rm in}\rangle$ between the initial and final vacuum states. Indeed, it determines the probability amplitude for the (spontaneous) decay of the vacuum specified on $\mathcal{J}^-$. The computation of Z is easy when the transformation is diagonal in energy: see, e.g., [2,8]. In the general case, however, as a result of the frequency mixing between in and out modes, the calculation of Z is less trivial. This generalization is presented in the Appendix.

It should also be noted that the Bogoliubov coefficients themselves are given by the following matrix elements:

$$\alpha^*_{\omega k}=\langle 0_{\rm in}|a^{\rm out}_\omega a^{\rm in\dagger}_k|0_{\rm in}\rangle,$$
$$-\beta^*_{\omega k}=\langle 0_{\rm in}|b^{\rm out\dagger}_\omega a^{\rm in\dagger}_k|0_{\rm in}\rangle. \quad (14)$$

However, it is not clear how to attribute a physical meaning to these equations. In particular, the relationship with the second one and pair creation amplitude is quite obscure. Indeed, the *probability* amplitude to obtain on $\mathcal{J}^+$ one pair of quanta of frequencies ω and ω' in the in vacuum is given by [see Eq. (A7) in the Appendix]

$$\langle 0_{\rm out}|a^{\rm out}_\omega b^{\rm out}_{\omega'}|0_{\rm in}\rangle=-\frac{1}{Z}\int_0^\infty dk\,\beta_{\omega k}\alpha^{-1}_{k\omega'}. \quad (15)$$

We shall return to these questions of interpretation in Sec. II C.

Instead of considering in-out matrix elements in Fock space, more attention has been put on the expectation values of (local) operators in a given initial state. The most studied object is probably the energy flux emitted by the mirror when the state of the field is the in vacuum. The motivations for this analysis are, first, its relevance for black hole radiation [2–11]; second, that its nonvanishing value is due to spontaneous pair creation, a specific feature of quantum field theory; and third, that this value can be computed either from using Eqs. (13) or from the properties of the Green's function of Φ.

Having at our disposal the Bogoliubov coefficients $\alpha_{\omega k}$ and $\beta_{\omega k}$, we start with the particle point of view. We consider the density energy of the emitted flux. The corresponding Hermitian operator is[1] $T_{VV}=\partial_V\Phi^\dagger\partial_V\Phi+\partial_V\Phi\partial_V\Phi^\dagger$. On the left of the mirror $[U>U_{\rm cl}(V)]$, using Eq. (13) and the first line of Eq. (10), the expectation value of the energy flux is

$$\langle T_{VV}\rangle\equiv\langle 0_{\rm in}|T_{VV}|0_{\rm in}\rangle-\langle 0_{\rm out}|T_{VV}|0_{\rm out}\rangle$$
$$=2\,{\rm Re}\Bigg\{\int\int_0^\infty d\omega\,d\omega'\frac{\sqrt{\omega\omega'}}{2\pi}\times\Bigg[e^{-i(\omega'-\omega)V}\Bigg(\int_0^\infty dk\,\beta^*_{\omega k}\beta_{\omega' k}\Bigg)-e^{-i(\omega'+\omega)V}\Bigg(\int_0^\infty dk\,\alpha^*_{\omega k}\beta_{\omega' k}\Bigg)\Bigg]\Bigg\}. \quad (16)$$

It should be noted that the subtraction of the out vacuum flux follows from the prescription of subtracting the contribution of the Minkowski vacuum. Indeed, by construction of the out modes, they coincide with the usual Minkowski modes on $\mathcal{J}^+$.

The total energy emitted to $\mathcal{J}^+$ is obtained from integrating $\langle T_{VV}\rangle$ over V. The integration eliminates the second term, which is due to interferences between states with different particle numbers. It gives

$$\langle H_V\rangle=\int_{-\infty}^{+\infty}dV\langle T_{VV}\rangle$$
$$=2\int_0^\infty d\omega\,\omega\int_0^\infty dk|\beta_{\omega k}|^2=2\int_0^\infty d\omega\,\omega\langle n_\omega\rangle. \quad (17)$$

One gets the usual relationship between the mean energy and mean number of particles, $\langle n_\omega\rangle=\int_0^\infty dk|\beta_{\omega k}|^2$, found on $\mathcal{J}^+$ (it equals the number of antiparticles). In this writing one sees that the nonvanishing character of $\langle H_V\rangle$ is due to the β coefficients which govern the vacuum decay; see Eq. (A6) in the Appendix.

We now reconsider the flux $\langle T_{VV}\rangle$ without making use of the Bogoliubov coefficients and with less emphasis on the notion of particle. This alternative method is based on the Wightman function evaluated in the in vacuum:

$$\langle 0_{\rm in}|\Phi^\dagger(U,V)\Phi(U',V')|0_{\rm in}\rangle$$
$$=\int_0^\infty dk\,\varphi^{\rm in}_k(U,V)\varphi^{\rm in*}_k(U',V'). \quad (18)$$

In terms of this function, using Eq. (5), the mean flux on $\mathcal{J}^+$ reads

$$\langle T_{VV}\rangle=2\lim_{V'\to V}[\langle 0_{\rm in}|\partial_V\Phi^\dagger\partial_{V'}\Phi|0_{\rm in}\rangle-\langle 0_{\rm out}|\partial_V\Phi^\dagger\partial_{V'}\Phi|0_{\rm out}\rangle]$$
$$=-\frac{1}{2\pi}\lim_{V'\to V}\partial_V\partial_{V'}[\ln|U_{\rm cl}(V')-U_{\rm cl}(V)|-\ln|V'-V|]$$
$$=\frac{1}{6\pi}\left\{\left(\frac{dU_{\rm cl}}{dV}\right)^{1/2}\partial_V^2\left[\left(\frac{dU_{\rm cl}}{dV}\right)^{-1/2}\right]\right\}$$

[1]The symmetrization is due to the fact that we deal with a complex field. Of course, in the DF model, particles and antiparticles equally contribute to $\langle T_{VV}\rangle$. This explains the overall factors of 2 in the next equations. We warn the reader that this equal contribution will not be necessarily found when considering partially transmitting mirrors.

$$= \frac{1}{24\pi}\left[\left(\frac{d^2U_{\rm cl}}{dV^2}\right)\left(\frac{dU_{\rm cl}}{dV}\right)^{-1}\right]^2 - \frac{1}{12\pi}\partial_V\left[\left(\frac{d^2U_{\rm cl}}{dV^2}\right)\left(\frac{dU_{\rm cl}}{dV}\right)^{-1}\right]. \quad (19)$$

Again, the subtraction of the out vacuum flux follows from the prescription of subtracting the contribution of the Minkowski vacuum. In this point splitting method, it is through that prescription that the notion of vacuum decay is reintroduced. Indeed, on $\mathcal{J}^+$, the above subtraction is equivalent to normal ordering with respect to out operators. [This is straightforwardly proved by using Eq. (10).] Moreover, the fact that $\langle T_{VV}\rangle$ and $\langle H_V\rangle$ vanish only for inertial trajectories,[2] i.e., when $\partial_V^2 U_{\rm cl}=0$, confirms that their nonvanishing character is due to the nonadiabaticity [24] of the scattering, a notion deeply rooted in the spontaneous creation of pairs of particles. In conclusion, a close examination of the point splitting method and that based on Bogoliubov coefficients reveals their complete agreement in flat space-time. In [25] this correspondence was extended to curved space-time by introducing Bogoliubov coefficients which are defined locally.

From Eq. (19) we learn that the energy flux is local in that it depends only on three derivatives of the trajectory $U_{\rm cl}(V)$ evaluated at the retarded time V (remember that we are on the left of the mirror). We shall see below that this locality is a consequence of dealing with a perfectly reflecting mirror for all frequencies.

Notice finally that in Eq. (16), the first term is positive definite and gives a positive total energy $\langle H_V\rangle$. Indeed, being a total derivative, the second term does not contribute to $\langle H_V\rangle$ when the trajectory is asymptotically inertial since $U_{\rm cl}(V)\sim V$ for asymptotically late and early V's. This might not be the case for trajectories which enter or leave the space through null infinities because of the infinite Doppler effect encountered asymptotically.

B. Partially transmitting mirrors

In this subsection we study partially transmitting (but still recoil-less) noninertial mirrors. We shall proceed in three steps. We first show that the scattering by a noninertial mirror is most simply described in terms of the wave functions which are eigenmodes of the proper time of the mirror. (We shall call them the proper-time modes.) When using these modes, the matrix relating the scattered modes to the initial ones is diagonal in the frequency, exactly as for the scattering of Minkowski modes by a mirror at rest. Second, we shall see that these modes are well adapted for introducing partially reflecting coefficients with arbitrary frequency-dependent phase and amplitude. Indeed, since this matrix is diagonal in the proper-time frequency, unitarity constrains its elements in a simple manner, frequency by frequency. Third, for both partial and total reflection, we shall see that the usual Bogoliubov coefficients, Eq. (9), relating the in and out Minkowski modes are simply obtained from this diagonal matrix.

To satisfy this program, we first need to construct the proper-time modes. To this end it is very useful to introduce new lightlike coordinates u, v such that the timelike coordinate $(u+v)/2=\tau$ is the proper time of the mirror and the spacelike one defined by $v-u/2=\rho$ is such that the trajectory reads $\rho=\rho_0=$const. These new coordinates are defined by two analytic functions $u(U)$ and $v(V)$ where U,V are the Minkowski lightlike coordinates. These functions are determined by the mirror trajectory $U_{\rm cl}(V)$ and the two conditions defining τ and ρ. Indeed, along the mirror's trajectory, the length element obeys

$$ds^2=\partial_V U_{\rm cl}(V)dV^2=\partial_U V_{\rm cl}(U)dU^2=dv^2=du^2=d\tau^2. \quad (20)$$

This gives

$$\frac{dv}{dV}=\sqrt{\partial_V U_{\rm cl}}, \quad \frac{du}{dU}=\sqrt{\partial_U V_{\rm cl}}. \quad (21)$$

One verifies that the Jacobians dv/dV and du/dU define a time-dependent boost since they satisfy $(du/dV)(du/dU)=1$ for all τ. The proper-time modes are then simply given by

$$\varphi_\lambda(u)=\frac{e^{-i\lambda u}}{\sqrt{4\pi|\lambda|}},$$

$$\varphi_\lambda(v)=\frac{e^{-i\lambda v}}{\sqrt{4\pi|\lambda|}}. \quad (22)$$

They form a complete basis on $\mathcal{J}^-$ and $\mathcal{J}^+$ since our condition that the trajectory emerge from i^- and finish on i^+ implies that the v and u axes cover those of V and U, respectively.

In the case of total reflection, the scattering along the mirror at $\rho=\rho_0$ is trivial. When using the conventions of the former subsection [Eqs. (6) and (11)], one has, on the left of the mirror,

$$\varphi_\lambda^{U,\rm in}(u,v)=\varphi_\lambda(u)-e^{2i\lambda\rho_0}\varphi_\lambda(v)=-e^{2i\lambda\rho_0}\varphi_\lambda^{V,\rm out}(u,v). \quad (23)$$

The new subscripts U and V indicate which side of $\mathcal{J}^-$ ($\mathcal{J}^+$) is the asymptotic support of the in (out) functions. We have introduced it in order to describe partial reflection which requires us to consider simultaneously both sides of the mirror. Using this notation, on the right of the mirror, one has

[2]For uniform acceleration, the two terms in Eq. (19) cancel each other, leading to a null flux. However, the Bogoliubov coefficients $\beta_{\omega k}$ do not vanish, thereby indicating that particles are produced. Moreover, when used in Eq. (17) they lead to a divergent integrated energy. To establish the compatibility of the null flux with this divergent result requires a regularization scheme. To obtain such a scheme is the main reason for considering the dynamical model of Sec. III.

$$\varphi_\lambda^{V,\mathrm{in}}(u,v)=\varphi_\lambda(v)-e^{-2i\lambda\rho_0}\varphi_\lambda(u)=-e^{-2i\lambda\rho_0}\varphi_\lambda^{U,\mathrm{out}}(u,v). \tag{24}$$

It will be useful to express these relations by a 2×2 matrix $\mathbf{S}_\lambda$ as

$$\varphi_\lambda^{i,\mathrm{out}}=S_\lambda^{ij}\varphi_\lambda^{j,\mathrm{in}}(\equiv a_\lambda^{j,\mathrm{in}}=S_\lambda^{ij}a_\lambda^{i,\mathrm{out}}). \tag{25}$$

At fixed λ, the indices of rows and columns i, j are the new subscript U or V. As usual, repeated indices are summed over. For total reflection, one has

$$\mathbf{S}_\lambda=\begin{pmatrix} 0 & -e^{+2i\lambda\rho_0} \\ -e^{-2i\lambda\rho_0} & 0 \end{pmatrix}, \tag{26}$$

We now consider partial reflection. When considering elastic reflection, the matrix $\mathbf{S}_\lambda$ relating in and out modes which generalizes Eq. (25) is unitary. [That is, we generalize total reflection in a restricted way since we keep both the linearity and the unitarity of Eq. (25).] Unitarity constrains the elements of $\mathbf{S}_\lambda$,

$$\mathbf{S}_\lambda=\begin{pmatrix} s_u e^{i\varphi_u} & -iRe^{i\varphi} \\ -iR' e^{i\varphi'} & s_v e^{i\varphi_v} \end{pmatrix}, \tag{27}$$

to obey

$$R=R', \quad s_u=s_v,$$

$$s_u^2+R^2=1, \quad \varphi'=\varphi_u+\varphi_v-\varphi. \tag{28}$$

(For simplicity of the expressions, we have not written the argument λ, but all variables should be understood as λ dependent.) Physically, R and s correspond to the reflection and transmission coefficients; i.e., when working in the rest frame of the mirror, the probability for an incident quantum of frequency λ to be reflected is R^2.

In what follows we impose $\varphi_u=\varphi_v=-\phi$, a condition which expresses that the transmitted part of the scattering is independent of the sign of the momentum. In anticipation of Sec. III, we point out that this equality is automatically satisfied when considering parity-invariant Hamiltonians (see [26], Chap. 3.4). In this case the matrix reads

$$\mathbf{S}_\lambda=e^{-i\phi}\begin{pmatrix} \sqrt{1-R^2} & -iRe^{i\theta} \\ -iRe^{-i\theta} & \sqrt{1-R^2} \end{pmatrix}. \tag{29}$$

In principle, the common phase $e^{-i\phi}$ could be reabsorbed in a redefinition of the modes. However, when using in and out modes conventionally defined, i.e., $\varphi_\lambda^{V,\mathrm{in}}(v)=\varphi_\lambda^{V,\mathrm{out}}(v)=\varphi_\lambda(v)$ of Eq. (22), the phase ϕ is unequivocally fixed. As we shall see in the next section, this convention is automatically used when considering interactions perturbatively. This is also the case in the DF model. Indeed, the limiting case of total reflection given in Eq. (26) is reached for $R\to 1$ and $\phi=\pi/2$ for all λ. One also finds that the other phase θ is related to the mirror location by $\theta=2\rho_0\lambda$.

To complete our second step, we should describe particles and antiparticles simultaneously. To this end, we group the in operators $(a_\lambda^{U,\mathrm{in}},a_\lambda^{V,\mathrm{in}},b_\lambda^{U,\mathrm{in}\dagger},b_\lambda^{V,\mathrm{in}\dagger})$ in a four-vector $a_\lambda^{\mu,\mathrm{in}}$ and the out operators $(a_\lambda^{U,\mathrm{out}},a_\lambda^{V,\mathrm{out}},b_\lambda^{U,\mathrm{out}\dagger},b_\lambda^{V,\mathrm{out}\dagger})$ in $a_\lambda^{\mu,\mathrm{out}}$. Similarly, we group their corresponding modes in the four-vectors $\varphi_\lambda^{\mu,\mathrm{in}}$ and $\varphi_\lambda^{\mu,\mathrm{out}}$. Since we work with a charged field, the modes associated with $b_\lambda^{i\dagger}$ might not be complex conjugates of those associated with a_λ^i (as is the case when dealing with a charged field in an electromagnetic field: see, e.g., Sec. 1.3 in [8]). Explicitly, in our case, the four modes are $(\varphi_\lambda^U,\varphi_\lambda^V,\bar{\varphi}_\lambda^{U}{}^*,\bar{\varphi}_\lambda^{V}{}^*)$ where $\bar{\varphi}_\lambda^i$ designate the two modes associated with the antiparticles operator b_λ^U and b_λ^V.

We then introduce the 4×4 matrix given by

$$\mathbf{S}_{\lambda\lambda'}=\delta(\lambda-\lambda')\begin{pmatrix} \mathbf{S}_\lambda & 0 \\ 0 & \overline{\mathbf{S}}_\lambda^* \end{pmatrix}, \tag{30}$$

where $\overline{\mathbf{S}}_\lambda$ is the scattering matrix for the antiparticles. Since $\mathbf{S}_{\lambda\lambda'}$ is block diagonal, unitarity constrains $\mathbf{S}_\lambda$ and $\overline{\mathbf{S}}_\lambda$ separately. $\mathbf{S}_{\lambda\lambda'}$ acts on the in four-vector as follows:

$$\varphi_\lambda^{\mu,\mathrm{out}}=S_{\lambda\lambda'}^{\mu\nu}\varphi_{\lambda'}^{\nu,\mathrm{in}}, \tag{31}$$

where continuous repeated indices are integrated from 0 to ∞ and discrete ones summed over the four components defined at fixed frequency. With these choices, the components of $\mathbf{S}_{\lambda\lambda'}$ are the Bogoliubov coefficients conventionally defined. By conventionally defined we mean the equations which generalize Eq. (13), i.e.,

$$a_{\lambda'}^{j,\mathrm{in}}=\alpha_{\lambda\lambda'}^{ij}a_\lambda^{i,\mathrm{out}}+\bar{\beta}_{\lambda\lambda'}^{ij}b_\lambda^{i,\mathrm{out}\dagger},$$

$$b_{\lambda'}^{j,\mathrm{int}\dagger}=\beta_{\lambda\lambda'}^{ij*}a_\lambda^{i,\mathrm{out}}+\bar{\alpha}_{\lambda\lambda'}^{ij*}b_\lambda^{i,\mathrm{out}\dagger}, \tag{32}$$

where the Bogoliubov coefficients $\alpha,\bar{\alpha},\beta,\bar{\beta}$ are now 2×2 matrices. By direct identification, one obtains

$$\alpha_{\lambda\lambda'}^{ij}=\langle\varphi_{\lambda'}^{j,\mathrm{in}}|\varphi_\lambda^{i,\mathrm{out}}\rangle=S_{\lambda\lambda'}^{ij},$$

$$\bar{\alpha}_{\lambda\lambda'}^{ij}=\langle\bar{\varphi}_{\lambda'}^{j,\mathrm{in}}|\bar{\varphi}_\lambda^{i,\mathrm{out}}\rangle=S_{\lambda\lambda'}^{i+2j+2*},$$

$$\beta_{\lambda\lambda'}^{ij}=\langle\bar{\varphi}_{\lambda'}^{j,\mathrm{in}}|\varphi_\lambda^{i,\mathrm{out}*}\rangle=S_{\lambda\lambda'}^{ij+2*},$$

$$\bar{\beta}_{\lambda\lambda'}^{ij}=\langle\varphi_{\lambda'}^{j,\mathrm{in}}|\bar{\varphi}_\lambda^{i,\mathrm{out}*}\rangle=S_{\lambda\lambda'}^{i+2j}. \tag{33}$$

When $\mathbf{S}_{\lambda\lambda'}$ is block diagonal in the sense of Eq. (30), one obviously has $\beta_{\lambda\lambda'}^{ij}=\bar{\beta}_{\lambda\lambda'}^{ij}=0$. In full generality, $\mathbf{S}_{\lambda\lambda'}$ satisfies unitarity in the following sense:

$$(S^\dagger)_{\lambda\lambda''}^{\mu\nu}S_{\lambda''\lambda}^{\nu\mu'}=\delta(\lambda-\lambda')\delta^{\mu\mu'}. \tag{34}$$

This equation generalizes Eqs. (10) to partially transmitting mirrors.

With Eqs. (26), (30), and (31), we have shown that scattering in the DF model is trivial when using proper-time modes. We have done more since Eqs. (31) and (33) apply to all partially transmitting mirrors governed by $\mathbf{S}_\lambda$ given by Eq. (29).

The last step consists in finding the relationship between $\mathbf{S}_{\lambda\lambda'}$ and the Bogoliubov coefficients between in and out Minkowski modes. This is simply achieved by introducing the 4×4 matrix which relates the (unscattered) Minkowski modes of frequency $k=-i\partial_t$ to the (unscattered) proper-time modes of frequency $\lambda=-i\partial_\tau$:

$$\phi_k^\mu=\mathcal{B}_{k\lambda}^{\mu\nu}\varphi_\lambda^\nu. \tag{35}$$

The elements of this matrix are given by

$$\mathcal{B}_{k\lambda}=\begin{pmatrix} \langle\varphi_\lambda^U|\phi_k^U\rangle & 0 & -\langle\varphi_\lambda^{U*}|\phi_k^U\rangle & 0\\ 0 & \langle\varphi_\lambda^V|\phi_k^V\rangle & 0 & -\langle\varphi_\lambda^{V*}|\phi_k^V\rangle\\ \langle\varphi_\lambda^U|\phi_k^{U*}\rangle & 0 & -\langle\varphi_\lambda^{U*}|\phi_k^{U*}\rangle & 0\\ 0 & \langle\varphi_\lambda^V|\phi_k^{V*}\rangle & 0 & -\langle\varphi_\lambda^{V*}|\phi_k^{V*}\rangle \end{pmatrix}=\begin{pmatrix} \alpha_{k\lambda}^{UU} & 0 & \beta_{k\lambda}^{UU*} & 0\\ 0 & \alpha_{k\lambda}^{VV} & 0 & \beta_{k\lambda}^{VV*}\\ \beta_{k\lambda}^{UU} & 0 & \alpha_{k\lambda}^{UU*} & 0\\ 0 & \beta_{k\lambda}^{VV} & 0 & \alpha_{k\lambda}^{VV*} \end{pmatrix}. \tag{36}$$

Since $\mathcal{B}_{k\lambda}$ relates unscattered modes, it is independent of the charge of the particle; hence, $\mathcal{B}_{k\lambda}^{11}\equiv\alpha_{k\lambda}^{UU}=\bar{\alpha}_{k\lambda}^{UU}\equiv\mathcal{B}_{k\lambda}^{33}$. The same equality applies to $\alpha_{k\lambda}^{VV}$, $\beta_{k\lambda}^{VV}$, and $\beta_{k\lambda}^{UU}$.

The important point for us is that $\mathcal{B}_{k\lambda}$ also relates the in Minkowski modes to the in proper-time modes and the out Minkowski modes to the out proper-time modes. Therefore the linear relation between in and out Minkowski modes is given by

$$\phi_\omega^{\mu,\mathrm{out}}=S_{\omega k}^{\mu\mu'}\phi_k^{\mu',\mathrm{in}}, \tag{37}$$

where

$$S_{\omega k}^{\mu\mu'}=\mathcal{B}_{\omega\lambda}^{\mu\nu}S_{\lambda\lambda'}^{\nu\nu'}(\mathcal{B}_{k\lambda'}^{-1})^{\nu'\mu'}. \tag{38}$$

Repeated indices are summed over, and the inverse of $\mathcal{B}$ is defined by

$$\varphi_\lambda^\nu=(\mathcal{B}_{k\lambda}^{-1})^{\nu\mu}\phi_k^\mu. \tag{39}$$

It is given by

$$\mathcal{B}_{k\lambda}^{-1}=\begin{pmatrix} \alpha_{k\lambda}^{UU*} & 0 & -\beta_{k\lambda}^{UU*} & 0\\ 0 & \alpha_{k\lambda}^{VV*} & 0 & -\beta_{k\lambda}^{VV*}\\ -\beta_{k\lambda}^{UU} & 0 & \alpha_{k\lambda}^{UU} & 0\\ 0 & -\beta_{k\lambda}^{VV} & 0 & \alpha_{k\lambda}^{VV} \end{pmatrix}. \tag{40}$$

Explicitly, using the dictionary (33) now applied to Minkowski modes, the four coefficients $S_{\omega k}^{1\nu}$ are

$$\alpha_{\omega k}^{UU}=\delta(\omega-k)-i\int_0^\infty d\lambda(\alpha_{\omega\lambda}^{UU}T_\lambda^{UU}\alpha_{k\lambda}^{UU*} \tag{41}$$
$$+\beta_{\omega\lambda}^{UU*}\bar{T}_\lambda^{UU*}\beta_{k\lambda}^{UU}),$$

$$\alpha_{\omega k}^{UV}=-i\int_0^\infty d\lambda(\alpha_{\omega\lambda}^{UU}T_\lambda^{UV}\alpha_{k\lambda}^{VV*}$$
$$+\beta_{\omega\lambda}^{UU*}\bar{T}_\lambda^{UV*}\beta_{k\lambda}^{VV}),$$

$$\beta_{\omega k}^{UU*}=i\int_0^\infty d\lambda(\alpha_{\omega\lambda}^{UU}T_\lambda^{UU}\beta_{k\lambda}^{UU*}+\beta_{\omega\lambda}^{UU*}\bar{T}_\lambda^{UU*}\alpha_{k\lambda}^{UU}),$$

$$\beta_{\omega k}^{UV*}=i\int_0^\infty d\lambda(\alpha_{\omega\lambda}^{UU}T_\lambda^{UV}\beta_{k\lambda}^{VV*}+\beta_{\omega\lambda}^{UU*}\bar{T}_\lambda^{UV*}\alpha_{k\lambda}^{VV}).$$

Similar equations give expressions for the other components of $S_{\omega k}^{\mu\nu}$. We have written $\mathbf{S}_\lambda$ as $\mathbf{S}_\lambda=\mathbf{1}-i\mathbf{T}_\lambda$ (and $\bar{\mathbf{S}}_\lambda=\mathbf{1}-i\bar{\mathbf{T}}_\lambda$) in order to extract the trivial part of the diagonal elements. This trivial part leads to the delta function in the first equation. The usefulness of this expression is that it will be easily related to the perturbative expressions we shall encounter in the next section.

Equations (41) are the central result of this section. They give the in-out overlaps of Minkowski modes in terms of the matrices $\mathbf{T}_\lambda,\bar{\mathbf{T}}_\lambda$ computed in the rest frame of the mirror and the overlaps between the free (unscattered) Minkowski and proper-time modes.

It is then easy to obtain the mean flux emitted by this partially transmitting noninertial mirror when the initial state of the field is the Minkowski vacuum. The same algebra which gave Eq. (16) now gives

$$\langle T_{VV}\rangle=\langle T_{VV}\rangle^{\mathrm{particle}}+\langle T_{VV}\rangle^{\mathrm{antiparticle}}, \tag{42}$$

where

$$\langle T_{VV}\rangle^{\mathrm{particle}}=\mathrm{Re}\Bigg\{\sum_{j=U,V}\int\int_0^\infty d\omega\, d\omega'\frac{\sqrt{\omega\omega'}}{2\pi}\times\Bigg[e^{-i(\omega'-\omega)V}\left(\int_0^\infty dk\,\beta_{\omega k}^{Vj*}\beta_{\omega' k}^{Vj}\right)-e^{-i(\omega'+\omega)V}\left(\int_0^\infty dk\,\bar{\alpha}_{\omega k}^{Vj*}\beta_{\omega' k}^{Vj}\right)\Bigg]\Bigg\}. \tag{43}$$

$\langle T_{VV}\rangle_{\mathrm{antiparticle}}$ is given by the same expression with $\bar{\alpha},\beta$ replaced by α,β. Here $\langle T_{VV}\rangle$ possesses the same structure as

Eq. (16). However, four kinds of coefficients α, β should be considered since we are dealing with *partial* reflection of *charged* particles.

When the scattering is independent of the energy and charge of the particles, i.e., when R and ϕ defined in Eq. (29) are independent of λ and when $\overline{\mathbf{S}}^*_\lambda = \mathbf{S}_{-\lambda}$, integration over λ can be trivially performed as it expresses the completeness of the φ_λ modes. In this case, as in the DF model, one has $\beta^{UU}_{\omega k} = \beta^{VV}_{\omega k} = 0$. One also finds that the emitted flux is simply

$$\langle T_{VV}\rangle = R^2 \langle T_{VV}\rangle_{\rm DF}, \tag{44}$$

where $\langle T_{VV}\rangle_{\rm DF}$ is the flux found in the DF model; see Eq. (16).

Instead, when R and ϕ depend on the energy and/or the charge, $\beta^{UU}_{\omega k}$ and $\beta^{VV}_{\omega k}$ will be, in general,[3] different from zero. In this case, one also loses the possibility of reexpressing the flux in terms of the derivatives of the trajectory as we did in Eq. (19). This can be understood from Eqs. (41): when expressing $\mathbf{T}_\lambda$ as a series in powers of λ, one would obtain for $\langle T_{VV}\rangle$ a nonlocal expression in V unless the series in λ stops after a finite number of terms.

C. Additional remarks

In this subsection, we relate the matrices $\mathbf{S}_{\lambda\lambda'}$ and $\mathbf{S}_{\omega k}$, which act linearly on in and out operators, to the conventional S matrix acting on multiparticle states in Fock space. With this identification we shall be able to relate the Bogoliubov coefficients, Eqs. (41), to *transition amplitudes* and not only to expectation values as in Eq. (43).

By definition [26], the action of this operator on states and operators is the following:

$$|0_{\rm in}\rangle = \hat{S}|0_{\rm out}\rangle,$$
$$a^{i,\rm out}_\lambda = \hat{S}^{-1} a^{i,\rm in}_\lambda \hat{S}, \quad b^{i,\rm out\dagger}_\lambda = \hat{S}^{-1} b^{i,\rm in\dagger}_\lambda \hat{S}. \tag{45}$$

Since we are dealing with elastic scattering, this operator contains exactly the same information as the matrices $\mathbf{S}_\lambda, \overline{\mathbf{S}}_\lambda$. Indeed, the block-diagonal character of Eq. (30) and the linearity of Eq. (31) tell us that $\hat{S}$ is the exponential of a quadratic form of the proper-time operators a_λ, b_λ:

$$\hat{S} = e^{-i(a^{i,\rm in}_\lambda s^{ij}_{\lambda\lambda'} a^{j,\rm in\dagger}_{\lambda'} - b^{i,\rm in\dagger}_\lambda \overline{s}^{ij}_{\lambda\lambda'} b^{j,\rm in}_{\lambda'})}. \tag{46}$$

Then straightforward algebra gives

$$\mathbf{s}_{\lambda\lambda'} = \delta(\lambda-\lambda')\begin{pmatrix} \phi & \arcsin(R)e^{i\theta} \\ \arcsin(R)e^{-i\theta} & \phi \end{pmatrix},$$
$$\overline{\mathbf{s}}_{\lambda\lambda'} = \mathbf{s}_{\lambda\lambda'}(\overline{R},\overline{\theta},\overline{\phi}), \tag{47}$$

[3] $\beta^{UU}_{\omega k}=0$ requires that $\overline{T}^{UU*}_\lambda = -T^{UU}_{-\lambda}$ for all $\lambda>0$ and similarly for the VV coefficients. In the next section, we shall see that the condition is satisfied for time-independent couplings with U(1) symmetry.

where R,θ,ϕ have been defined in Eq. (29) and $\overline{R},\overline{\theta},\overline{\phi}$ are defined in the same way from $\overline{\mathbf{S}}_\lambda$. We note that in the DF model, i.e., in the limit of perfect reflection, $\mathbf{s}_{\lambda\lambda'}$ is given by

$$\mathbf{s}^{\rm DF}_{\lambda\lambda'} = \delta(\lambda-\lambda')\left(\frac{\pi}{2}\right)\begin{pmatrix} 1 & e^{i\theta} \\ e^{-i\theta} & 1 \end{pmatrix} = \overline{\mathbf{s}}^{\rm DF}_{\lambda\lambda'}. \tag{48}$$

Although the configurations on the left and right of the mirror completely decouple, the S matrix $\hat{S}$ treats both sides simultaneously.

To anticipate the expression of $\hat{S}$ in terms of Minkowski operators which will mix creation and destruction operators, it is a convenient to rewrite Eq. (46) in term of the four-vector $a^{\mu,\rm in}_\lambda$,

$$\hat{S} = e^{-i(a^{\mu,\rm in}_\lambda s^{\mu\nu}_{\lambda\lambda'} a^{\nu,\rm in\dagger}_{\lambda'})}, \quad \text{with} \quad (s^{\mu\nu}_{\lambda\lambda'}) = \begin{pmatrix} s^{ij}_{\lambda\lambda'} & 0 \\ 0 & -\overline{s}^{ij}_{\lambda\lambda'} \end{pmatrix}. \tag{49}$$

To obtain the expression of $\hat{S}$ in terms of the Minkowski operators $a^{i,\rm in}_k, b^{i,\rm in}_k$, it suffices to use the matrix $\mathcal{B}_{\omega\lambda}$ to replace proper-time operators by Minkowski ones. Explicitly, one obtains

$$\hat{S} = \exp[-i(a^{\mu,\rm in}_\omega s^{\mu\nu}_{\omega\omega'} a^{\nu,\rm in\dagger}_{\omega'})],$$

with

$$s^{\mu\nu}_{\omega\omega'} = \mathcal{B}^{\mu\mu'}_{\omega\lambda} s^{\mu'\nu'}_{\lambda\lambda'} (\mathcal{B}^\dagger_{\omega'\lambda'})^{\nu\nu'}. \tag{50}$$

Formally, $\hat{S}$ provides the answer to all questions concerning asymptotic states and expectation values. For instance, the probability amplitude governing the (Minkowski) vacuum decay, Eq. (A6), is simply

$$Z^{-1} = \langle 0_{\rm out}|0_{\rm in}\rangle = \langle 0_{\rm in}|\hat{S}|0_{\rm in}\rangle. \tag{51}$$

Similarly, the probability amplitude for an initial quantum of momentum k to be scattered and for no pair to be created is

$$\langle 0_{\rm out}|a^{i,\rm out}_\omega a^{j,\rm in\dagger}_k|0_{\rm in}\rangle = \langle 0_{\rm in}|a^{i,\rm in}_\omega \hat{S} a^{j,\rm in\dagger}_k|0_{\rm in}\rangle = \frac{1}{Z}(\alpha^{-1})^{ji}_{k\omega}. \tag{52}$$

The last equality is easily obtained by using Eq. (32) to express $a^{i,\rm out}_\omega$ in terms of $a^{j,\rm in}_k$ and $b^{j,\rm out\dagger}_\omega$. In the same way, the Bogoliubov coefficient β is related to the probability amplitude to find a pair of out quanta in the in vacuum by

$$\beta^{ij}_{\omega k}(\overline{\alpha}^{-1})^{ji'}_{k\omega'} = -\frac{\langle 0_{\rm out}|a^{i,\rm out}_\omega b^{i',\rm out}_{\omega'}|0_{\rm in}\rangle}{\langle 0_{\rm out}|0_{\rm in}\rangle}$$
$$= -\frac{\langle 0_{\rm in}|a^{i,\rm in}_\omega b^{i',\rm in}_{\omega'}\hat{S}|0_{\rm in}\rangle}{\langle 0_{\rm in}|\hat{S}|0_{\rm in}\rangle}. \tag{53}$$

In this, we recover Eq. (15). It should be stressed that these relations *determine* the physical interpretation of the overlaps α, β given in Eqs. (41). In fact, the second-quantized framework was never used to obtain Eq. (41): only the linearity of the relations and the orthonormal character of the proper-time and Minkowski-mode bases were exploited.

The physical interpretation of α, β is the following: to first order in the transfer matrix $\mathbf{T}_\lambda$, α (β) divided by Z gives the probability amplitude to scatter a quantum (to produce a pair of quanta), since $\alpha^{-1}\simeq 1+iT$ ($\beta\simeq -iT$). Upon considering higher-order terms in $\mathbf{T}_\lambda$, one loses the simplicity of the relationship so as to get the above equations. The simple relation in the linear regime will be nicely confirmed in the next section when using perturbation theory. We shall see in particular that division by Z corresponds to the usual restriction of keeping only the connected graphs engendered by the development of $\hat{S}=Te^{-ig\int dt\, H}$ in powers of g. We shall further comment on these aspects at the end of Sec. III.

III. SELF-INTERACTING MODEL

In this section, following [28,29,10], we introduce a model based on self-interactions which derives from an action principle. In the first part, we consider time-independent couplings. In this case, resumming the Born series leads, as in Eq. (30), to diagonal matrices in the proper-time energy λ with parameters R and ϕ, which depend on λ according to the number of derivatives in the interaction Hamiltonian. The important difference is that causality is now built in, as follows from interactions governed by an action. This model will also be generalized by considering a thick mirror with a nonzero width. Using a perturbative approach, we shall see that the thickness acts as a UV cutoff.

In the second part, we work with time-dependent couplings. We shall work perturbatively up to second order in the interactions. The novelty concerns the transients induced by the switching on and off of the coupling.

A. Scattering with *g* constant

To exploit the fact that the coupling is τ independent, it is convenient to work with the coordinates (τ, ρ) in which the mirror is at rest. In these coordinates, the interaction Lagrangian reads

$$L_{\rm int}=g\int\int_{\infty}^{+\infty}d\tau\, d\rho f(\rho)J(\Phi(\tau,\rho),\Phi^\dagger(\tau,\rho)). \qquad (54)$$

Here g is the coupling parameter, and f is a real function which specifies the thickness of the mirror and which is normalized as $\int_{-\infty}^{+\infty}d\rho\, f(\rho)=1$. Also, J is a Hermitian operator which is quadratic in the complex field. We shall consider three different cases: $\Phi^\dagger\Phi+\Phi\Phi^\dagger$, $\Phi^\dagger i\overleftrightarrow{\partial}_\tau\Phi$, and $\partial_\tau\Phi^\dagger\partial_\tau\Phi+\partial_\tau\Phi\partial_\tau\Phi^\dagger$. In the following equations, we shall present the details only with the second expression. At the end of the derivation, we shall give the final results for the two other cases.

Given Eq. (54), Eq. (1) is now replaced by

$$(\partial_\tau^2-\partial_\rho^2)\Phi(\tau,\rho)=gf(\rho)2i\partial_r\Phi. \qquad (55)$$

Being linear, the solution can be expressed as

$$\Phi(\tau,\rho)=\Phi^{\rm in}(\tau,\rho)+g\int\int_{-\infty}^{+\infty}d\tau'\, d\rho' \times G^{\rm ret}(\tau,\rho;\tau',\rho')f(\rho')2i\partial_{\tau'}\Phi(\tau',\rho')$$
$$=\Phi^{\rm out}(\tau,\rho)+g\int\int_{-\infty}^{+\infty}d\tau'\, d\rho' \times G^{\rm adv}(\tau,\rho;\tau',\rho')f(\rho')2i\partial_{\tau'}\Phi(\tau',\rho'), \qquad (56)$$

in terms of the homogeneous solution $\Phi^{\rm in}$ ($\Phi^{\rm out}$) which determines the initial (final) data. The retarded and advanced Green's functions are defined, as usual, by

$$G^{\rm ret}(\tau,\rho;\tau',\rho')=\int\int_{-\infty}^{+\infty}d\lambda\, dl\,\frac{1}{4\pi^2}\,\frac{e^{-i\lambda(\tau-\tau')+il(\rho-\rho')}}{l^2-(\lambda+i\epsilon)^2} \quad (=0\ \text{ for }\ \tau'>\tau),$$

$$G^{\rm adv}(\tau,\rho;\tau',\rho')=\int\int_{-\infty}^{+\infty}d\lambda\, dl\,\frac{1}{4\pi^2}\,\frac{e^{-i\lambda(\tau-\tau')+il(\rho-\rho')}}{l^2-(\lambda-i\epsilon)^2} \quad (=0\ \text{ for }\ \tau'<\tau). \qquad (57)$$

To exploit the time independence of the coupling g, we work at fixed energy with

$$\varphi_\lambda(\rho)=\int_{-\infty}^{+\infty}d\tau\frac{1}{2\pi}\Phi(\tau,\rho)e^{i\lambda\tau}. \qquad (58)$$

In a Fourier transform, Eqs. (56) give

$$\varphi_\lambda(\rho)=\varphi_\lambda^{\rm in}(\rho)+ig\int_{-\infty}^{+\infty}d\rho'\, f(\rho')\varphi_\lambda(\rho')e^{i\lambda|\rho-\rho'|}$$
$$=\varphi_\lambda^{\rm out}(\rho)-ig\int_{-\infty}^{+\infty}d\rho'\, f(\rho')\varphi_\lambda(\rho')e^{-i\lambda|\rho-\rho'|}. \qquad (59)$$

These equations have been obtained by using

$$\int_{-\infty}^{+\infty}dl\,\frac{e^{il(\rho-\rho')}}{l^2-(\lambda\pm i\epsilon)^2}=\frac{\pm 2i\pi}{2(\lambda\pm i\epsilon)}e^{\pm i\lambda|\rho-\rho'|}, \qquad (60)$$

and by having taken the limit $\epsilon\to 0$.

We now decompose the quantized modes $\varphi_\lambda^{\rm in}$ in terms of creation and destruction operators:

$$\varphi_\lambda^{\rm in}(\rho)=\frac{1}{\sqrt{4\pi\lambda}}(a_\lambda^{U,\rm in}e^{i\lambda\rho}+a_\lambda^{V,\rm in}e^{-i\lambda\rho}) \quad (\text{for }\ \lambda>0)$$
$$=\frac{1}{\sqrt{4\pi|\lambda|}}(b_{|\lambda|}^{U,\rm in\dagger}e^{-i|\lambda|\rho}+b_{|\lambda|}^{V,\rm in\dagger}e^{i|\lambda|\rho}) \quad (\text{for }\ \lambda<0). \qquad (61)$$

We do the same for the out modes. Then, for $f(\rho)=\delta(\rho-\rho_0)$, Eqs. (59) give

$$\begin{pmatrix} a_\lambda^{U,\text{out}} \\ a_\lambda^{V,\text{out}} \end{pmatrix} = \frac{1}{1-ig} \begin{pmatrix} 1 & ige^{-2i\lambda\rho_0} \\ ige^{2i\lambda\rho_0} & 1 \end{pmatrix} \begin{pmatrix} a_\lambda^{U,\text{in}} \\ a_\lambda^{V,\text{in}} \end{pmatrix}. \quad (62)$$

We recover the linear structure of $\mathbf{S}_\lambda$ in Eq. (25). Since the unitarity of $\mathbf{S}_\lambda$ provides $a_\lambda^{i,\text{out}} = S_\lambda^{ij*} a_\lambda^{j,\text{in}}$, when using the definitions of Eq. (29), we obtain

$$R = \frac{g}{\sqrt{1+g^2}}, \quad \phi = \arctan(g), \quad \theta = 2\lambda\rho_0. \quad (63)$$

In the strong-coupling limit (i.e., for $g \to +\infty$), one obtains the total reflection (26) in a λ-independent manner. This is a special feature of the coupling $J = \Phi^\dagger i \overleftrightarrow{\partial}_\tau \Phi$, which is associated with a dimensionless g.

This analysis can be repeated with the two other operators previously defined. The presence or absence of derivatives in J modifies the IR or UV behavior of R. For $J = \Phi^\dagger \Phi + \Phi \Phi^\dagger$, one obtains [28]

$$R_\lambda = \frac{g/\lambda}{\sqrt{1+g^2/\lambda^2}}, \quad \phi_\lambda = \arctan(g/\lambda), \quad \theta = 2\lambda\rho_0. \quad (64)$$

In this case, the mirror is totally reflecting in the IR. This leads to strong IR divergences when considering time-dependent coupling g. On the contrary, when $J = \partial_\tau \Phi^\dagger \partial_\tau \Phi + \partial_\tau \Phi \partial_\tau \Phi^\dagger$, we get

$$R_\lambda = \frac{g\lambda}{\sqrt{1+g^2\lambda^2}}, \quad \phi_\lambda = \arctan(g\lambda), \quad \theta = 2\lambda\rho_0. \quad (65)$$

In this case, the mirror is transparent in the IR limit. This useful property will be exploited in Sec. IV.

We notice that the transfer matrix $\mathbf{T}_\lambda$ can be expressed in a general way according to the number n of derivatives ∂_τ in the interaction term:

$$\mathbf{T}_\lambda = \frac{-g\lambda^{n-1}}{1-ig\lambda^{n-1}A_\epsilon} \begin{pmatrix} 1 & e^{2i\lambda\rho_0} \\ e^{-2i\lambda\rho_0} & 1 \end{pmatrix}. \quad (66)$$

In this expression, we have not taken the limit $\epsilon \to 0$ in using Eq. (60). The function $A_\epsilon = \lambda/(\lambda + i\epsilon)$ determines the analytical properties of $\mathbf{T}_\lambda$ in the complex λ plane. The specification of the pole of A_ϵ follows from that of G^{ret} in Eq. (57). It guarantees that *causality* will be respected [28]. This crucial ingredient was missing in Sec. II B wherein the matrix $\mathbf{T}_\lambda$ can be chosen from the outset. In that kinematic framework, the analytical properties should be imposed by hand if one wishes to implement causality. On the contrary, in the present case causality follows from the Heisenberg equations (56).

Equations (59) and (61) also determine the relation between the antiparticle in and out operators $b_\lambda^{i\dagger}$. By direct computation one finds $\overline{\mathbf{T}}_\lambda^* = -\mathbf{T}_{-\lambda}$. This is precisely the condition which gives $\beta_{\omega k}^{UU} = \beta_{\omega k}^{VV} = 0$; see footnote 3. When using $\mathbf{T}_\lambda, \overline{\mathbf{T}}_\lambda$ in Eqs. (41), we obtain the Bogoliubov coefficients relating inertial modes. And from these coefficients, one gets the mean value of the energy flux T_{VV} as in Eq. (43), but with causality built in.

We now study the case of a thick mirror with J given by $\Phi^\dagger i \overleftrightarrow{\partial}_\tau \Phi$. To display the effects of $f(\rho)$ in Eq. (54), it is convenient to work with the (spatial) Fourier components. Equation (59) becomes

$$\varphi_{\lambda,l} = \varphi_{\lambda,l}^{\text{in}} - \frac{2g\lambda}{(\lambda+i\epsilon)^2 - l^2} \int_{-\infty}^{+\infty} dl' \, f_{l-l'} \varphi_{\lambda,l'}$$
$$= \varphi_{\lambda,l}^{\text{out}} - \frac{2g\lambda}{(\lambda-i\epsilon)^2 - l^2} \int_{-\infty}^{+\infty} dl' \, f_{l-l'} \varphi_{\lambda,l'}. \quad (67)$$

For an arbitrary window function f, these equations do not lead to analytic relations between asymptotic in and out fields. Therefore, to estimate the effects of $f(\rho)$, we use perturbation theory. To first order in g, we get

$$\mathbf{T}_\lambda^f = -g \begin{pmatrix} 1 & 2\pi f_{2\lambda}^* \\ 2\pi f_{2\lambda} & 1 \end{pmatrix}. \quad (68)$$

For a normalized Gaussian function f centered on ρ_0, the nondiagonal terms which determine the reflection probability are $ge^{\pm 2i\lambda\rho_0} e^{-2\lambda^2\sigma^2}$. Therefore, σ, the spread of the mirror, reduces the reflection of high frequencies: for $\lambda \gg 1/\sigma$, the mirror is completely transparent (this is also true for the two other J's).

B. Scattering with g time dependent

In this subsection, the coupling parameter is a function of the proper time $g(\tau) = gf(\tau)$ where $f(\tau)$ is normalized by $\int_{-\infty}^{+\infty} d\tau \, f(\tau) = 2T$, with $2T$ the proper-time lapse during which the interactions are turned on. Unlike what we had in the former subsection, resumming the Born series is no longer possible since the time dependence of the coupling destroys the decoupling of the equations into sectors at fixed frequency λ. Thus we shall work perturbatively: all quantities will be evaluated up to second order in g. In fact, we meet a situation analogous of that of a thick mirror which mixes different momenta.

We first remind the reader that in the interacting picture, the operator Φ evolves freely, i.e., with $g=0$: it obeys Eq. (1) and not Eq. (55). Therefore the in operators a_ω, b_ω specified at $t = -\infty$ coincide with the out operators and are equal to the usual Minkowski operators. Hence they define the (Minkowski) vacuum $|0\rangle$. Instead, the states evolve through the action of the time-ordered operator:

$$|\Psi(t=+\infty)\rangle = Te^{iL} |\Psi(t=-\infty)\rangle, \quad (69)$$

where $L = g \int d\tau \, f(\tau) J$ engenders self-interactions. Since the trajectory is time like, T, the time ordering with respect to the Minkowski time t, is equivalent to that of the proper time τ.

To make contact with Sec. II, we work in this section with the state $|\Psi_0(t)\rangle$ which is equal to $|0\rangle$ for $t = \tau = -\infty$. When expressing its final value in the basis of the unperturbed states, i.e., the states which would have been stationary in the absence of interactions $(g=0)$, we get

$$|\Psi_0(\tau=+\infty)\rangle = |0\rangle + \sum_{i,j} \int\int_0^\infty d\omega\, d\omega' (B^{ij}_{\omega\omega'} + C^{ij}_{\omega\omega'})|\omega\omega'\rangle_{ij}, \tag{70}$$

where

$$B^{ij}_{\omega\omega'} = ig\langle 0|a^i_\omega b^j_{\omega'}\left(\int_{-\infty}^{+\infty} d\tau\, f(\tau)J(\tau)\right)|0\rangle,$$

$$|\omega\omega'\rangle_{ij} \equiv a^{i\dagger}_\omega b^{j\dagger}_{\omega'}|0\rangle, \tag{71}$$

$$C^{ij}_{\omega\omega'} = -g^2\langle 0|a^i_\omega b^j_{\omega'} \times \left(\int_{-\infty}^{+\infty} d\tau \int_{-\infty}^{\tau} d\tau'\, f(\tau)f(\tau')J(\tau)J(\tau')\right)|0\rangle_c. \tag{72}$$

We have limited the expansion in g to these three terms since we shall compute the energy-momentum tensor up to g^2 terms only. As before, i,j denote the U, V sectors and ω,ω' Minkowski energies. The symbol $\langle\ \rangle_c$ means that only the *connected* part of the expectation value is kept. This restriction follows from the fact that the contribution of the disconnected graphs cancels out since they also appear in the denominator of the matrix elements; see, e.g., [26].

Using Eq. (70), the expectation value of T_{VV} is given by

$$\begin{aligned}\langle T_{VV}\rangle &= \langle\Psi_0(\tau=+\infty)|T_{VV}|\Psi_0(\tau=+\infty)\rangle_c\\ &= \mathrm{Re}\Bigg\{\sum_j \int\int_0^\infty d\omega\, d\omega' \frac{\sqrt{\omega\omega'}}{2\pi}\\ &\quad\times\Bigg[e^{-iV(\omega'-\omega)}\int_0^\infty dk(B^{Vj}_{\omega k}{}^* B^{Vj}_{\omega' k} + \bar{B}^{Vj}_{\omega k}{}^*\bar{B}^{Vj}_{\omega' k})\\ &\quad - e^{-iV(\omega'+\omega)}(B^{VV}_{\omega\omega'} + \bar{B}^{VV}_{\omega\omega'} + C^{VV}_{\omega\omega'} + \bar{C}^{VV}_{\omega\omega'})\Bigg]\Bigg\},\end{aligned} \tag{73}$$

where $\bar{B}^{ij}_{\omega k}$ and $\bar{C}^{VV}_{\omega\omega'}$ are related to the unbarred quantities by inverting particle and antiparticle operators: thus, $\bar{B}^{ij}_{\omega\omega'} = B^{ji}_{\omega'\omega}$ and $\bar{C}^{VV}_{\omega\omega'} = C^{VV}_{\omega'\omega}$.

Since the integral of the second term in Eq. (73) vanishes, and since barred and unbarred quantities differ at most by a phase, the total energy received on the V part of $\mathcal{J}^+$ is

$$\langle H_V\rangle \equiv \int_{-\infty}^{+\infty} dV\langle T_{VV}\rangle_c = 2\sum_j \int_0^\infty d\omega\, \omega \int_0^\infty dk |B^{Vj}_{\omega k}|^2. \tag{74}$$

Hence only the B terms contribute to the energy as the β terms did in Eq. (17).

In order to compute the local properties of the flux, we need to compute the second term of Eq. (73). To this end we decompose $C^{VV}_{\omega\omega'}$ into two parts:

$$C^{VV}_{\omega\omega'} = R^{VV}_{\omega\omega'} - \langle 0|a^V_\omega b^V_{\omega'}\mathcal{D}|0\rangle_c, \tag{75}$$

where

$$R^{VV}_{\omega\omega'} = -\frac{1}{2}\langle 0|a^V_\omega b^V_{\omega'} LL|0\rangle_c, \tag{76}$$

$$\mathcal{D} = \frac{g^2}{2}\left(\int_{-\infty}^{+\infty} d\tau \int_{-\infty}^{+\infty} d\tau'\, f(\tau)f(\tau')\epsilon(\tau-\tau')J(\tau)J(\tau')\right) \tag{77}$$

and $\epsilon(\tau-\tau') = \theta(\tau-\tau') - \theta(\tau'-\tau)$. Then $\langle T_{VV}\rangle_{\mathcal{D}}$, the contribution of $\mathcal{D}$ to $\langle T_{VV}\rangle$, enjoys the following properties (see Appendix A in [17] for a similar analysis applied to a two-level atom coupled to radiation). First, it carries no energy. This is obvious since it is built with terms which all contain $e^{iV(\omega+\omega')}$. Second, it vanishes for $f(\tau) = cst$. This can be understood from the fact that the time ordering properties can be encoded in the analytical properties of the matrix $\mathbf{T}_\lambda$ which is diagonal in λ; see Eq. (66). This means that this term modifies the shape of the transients related to the switching on and off of the interaction, but without affecting their energy content. In the rest of the paper, we shall therefore ignore this term.

We now compute $R^{VV}_{\omega\omega'}$. Since only the connected part should be kept, we can insert the following operator between the two operators L in Eq. (76):

$$\sum_{i,j} \int_0^\infty dk \int_0^\infty dk'\, a^{i\dagger}_k b^{j\dagger}_{k'}|0\rangle\langle 0|a^i_k b^j_{k'}. \tag{78}$$

Grouping together, as in Eq. (73), the first- and second-order contributions in g, we get

$$B^{VV}_{\omega\omega'} + \bar{B}^{VV}_{\omega\omega'} + R^{VV}_{\omega\omega'} + \bar{R}^{VV}_{\omega\omega'} = \sum_i \int_0^\infty dk(\bar{A}^{Vj}_{\omega k}{}^* B^{Vj}_{\omega' k} + A^{Vj}_{\omega' k}{}^*\bar{B}^{Vj}_{\omega k}), \tag{79}$$

with

$$A^{ij}_{\omega k}{}^* = \langle 0|a^i_\omega(1+iL)a^{j\dagger}_k|0\rangle_c,$$

$$\bar{A}^{ij}_{\omega k}{}^* = \langle 0|b^i_\omega(1+iL)b^{j\dagger}_k|0\rangle_c. \tag{80}$$

Hence we find that $\langle T_{VV}\rangle$ is given by Eq. (42) with

$$\begin{aligned}\langle T_{VV}\rangle^{\mathrm{particle}} &= \mathrm{Re}\sum_j \int_0^\infty d\omega \int_0^\infty d\omega' \frac{\sqrt{\omega\omega'}}{2\pi}\\ &\quad\times\Bigg[e^{-iV(\omega'-\omega)}\left(\int_0^\infty dk\, B^{Vj}_{\omega k}{}^* B^{Vj}_{\omega' k}\right)\\ &\quad - e^{-iV(\omega'+\omega)}\left(\int_0^\infty dk\, \bar{A}^{Vj}_{\omega k}{}^* B^{Vj}_{\omega' k}\right)\Bigg].\end{aligned} \tag{81}$$

$\langle T_{VV}\rangle_{\mathrm{antiparticle}}$ is given by the same expression with $\bar{A}^{Vj}_{\omega k}, B^{Vj}_{\omega k}$ replaced by $A^{Vj}_{\omega k}, \bar{B}^{Vj}_{\omega k}$.

Thus, to second order in g, we recover the structure of Eq. (43), which gives the flux emitted by a partially transmitting mirror. The Bogoliubov coefficients $\alpha_{\omega k}^{Vj}$ and $\beta_{\omega k}^{Vj}$ have been replaced by the transition amplitudes $A_{\omega k}^{Vj}$ and $B_{\omega k}^{Vj}$. In this, we recover the correspondence of Eqs. (52) and (53) when considered to first order in the transfer matrix $\mathbf{T}_\lambda$. This is not surprising since the evolution operator Te^{iL}, which defines $A^*_{\omega k}$ and $B_{\omega k}$ given in Eqs. (80) and (71), is, *by definition*, the operator $\hat{S}$ of Eq. (45).

This correspondence is nicely illustrated in the case where $g(\tau)=g$ and $J=\Phi^\dagger i\overleftrightarrow{\partial}_\tau\Phi$. In this case, to order g, but whatever is the mirror's trajectory $U=U_{\rm cl}(V)$, one has the following identities:

$$A_{\omega k}^{VU}=g\alpha_{\omega k}, \quad B_{\omega k}^{VU}=g\beta_{\omega k}, \tag{82}$$

where $\alpha_{\omega k}$ and $\beta_{\omega k}$ are the overlaps, Eq. (9), computed in the DF model. These relations establish that $\alpha_{\omega k}$ and $\beta_{\omega k}$ should be viewed as transition amplitudes. This is important for the following reason. It implies that the momentum transfers to the mirror (which have been neglected so far) associated with the transitions described by A and B are, respectively, $\hbar(k+\omega)$ and $\hbar(-k+\omega)$. This fact imposes limitations when considering the emission of ultrahigh (trans-Planckian) frequencies since neglecting the momentum transfers requires $\hbar\omega \ll M$, where M is the mass of the mirror [10]. Thus, when high-frequency quanta are emitted, the validity of the predictions obtained with a recoil-less model *must* be questioned [11].

IV. APPLICATIONS

The aim of this section is to illustrate the usefulness of the dynamical model in which one can switch on and off the coupling between the mirror and radiation field. First, we analyze the properties of the energy flux associated with the switching on and off when the mirror is at rest ($z=0$) and in Minkowski vacuum. As expected, we shall see that the flux is localized in the transitory periods where the coupling is turned on or off. Moreover, the mean frequency emitted is given by the switching rate of the coupling.

Second, we generalize this analysis by replacing the Minkowski vacuum by a thermal bath. Then we use the well-known parallel between inertial systems in a thermal bath and uniformly accelerated systems in vacuum to obtain (for the first time) a regularized expression of the flux emitted by a uniformly accelerated mirror.

A. Transients in vacuum

We first focus on the frequency content of the transients. For an inertial mirror at rest at $z=0$ in Minkowski vacuum, the transition amplitudes A and B of Eqs. (80) and (71) can be expressed in terms of the Fourier transforms of $f(t)$:

$$f_\omega=\frac{1}{2\pi}\int dt\, f(t)e^{i\omega t}. \tag{83}$$

To order g, we obtain

$$A_{\omega k}^{ij*}=\delta(\omega-k)\delta^{ij}+igf_{\omega-k}\frac{j(\omega,k)}{\sqrt{\omega k}}, \tag{84}$$

$$B_{\omega k}^{ij}=igf_{\omega+k}\frac{j(\omega,-k)}{\sqrt{\omega k}}, \tag{85}$$

where

$$j(\omega,k)=\begin{cases} 1 & \text{for } \Phi^\dagger\Phi+\Phi\Phi^\dagger, \\ \omega+k & \text{for } \Phi^\dagger i\overleftrightarrow{\partial}_t\Phi, \\ \omega k & \text{for } \partial_t\Phi^\dagger\partial_t\Phi+\partial_t\Phi\partial_t\Phi^\dagger. \end{cases} \tag{86}$$

Thus, to order g^2, the mean number of V particles of energy ω is given by

$$\langle N_\omega^V\rangle=2\sum_j\int_0^\infty dk|B_{\omega k}|^2. \tag{87}$$

The factor of 2 arises from the fact that it is equally probable to emit a UV or a VV pair of quanta of energy ω and k.

To further analyze the transients associated with the switching on and off of the coupling to the mirror, we shall work with the function

$$f(t)=\frac{1}{2}\left[\tanh\left(\frac{t+T}{\Delta}\right)-\tanh\left(\frac{t-T}{\Delta}\right)\right]. \tag{88}$$

It is almost constant during a lapse of time $2(T-\Delta)$ centered about $t=0$, and the time intervals of the switching on and off are $\simeq 4\Delta$. In the limit $\Delta\to 0$, f tends to the square window $[\theta(t+T)-\theta(t-T)]/2$. The Fourier components of f are

$$f_\omega=\frac{\Delta}{2}\frac{\sin(\omega T)}{\sinh(\omega\pi\Delta/2)}. \tag{89}$$

One sees that the UV behavior is exponentially damped by Δ. On the contrary, in the IR, $f_\omega\to T/\pi$, as expected since the coupling lasts $2T$.

When considering the first two cases of $j(\omega,k)$ of Eq. (86), this last observation implies that the mean number $\langle N_\omega\rangle$ is ill defined since the integral over k in Eq. (87) diverges in the IR. Therefore, to obtain well-defined expressions, we shall work with the third case. In this case, one has

$$\langle N_\omega^V\rangle=\frac{g^2\Delta^2}{2}\omega\int_0^\infty d\omega'\,\omega'\frac{\sin^2[(\omega+\omega')T]}{\sinh^2[(\omega+\omega')\pi\Delta/2]}. \tag{90}$$

It is perhaps appropriate to discuss the condition on the (dimensionful) coupling constant g which guarantees the validity of Eq. (87), which follows from a perturbative treatment limited to order g^2. The condition is that the mean number of quanta per quantum cell (which is equal to $\langle N_\omega\rangle d\omega\simeq\langle N_\omega\rangle\pi/T$ in the limit $\omega T\gg 1$) be well approximated by Eq. (87). This requires that the probability to obtain two quanta in a cell is negligible with respect to that of obtaining one. This translates mathematically by $g^2\ll T\Delta$ in the limit of interest $T/\Delta\gg 1$, i.e., when the flat plateau is much longer that the slopes. The condition $g^2\ll T\Delta$ means that the limit $T\to\infty$ can be safely taken. Instead, the limit

$\Delta\to 0$ is more delicate. A sufficient condition consists in working at fixed $\tilde{g}^2 \ll 1$, where $\tilde{g} = g(T\Delta)^{-1/2}$. A stronger condition is to impose that the total number of particle emitted, $\int_0^\infty \langle N_\omega^V \rangle d\omega$, be finite in the limit $\Delta\to 0$. In this case, $\bar{g} = g/\Delta$ should be held fixed.

When studying Eq. (90), one first notices that in the limit $T\to\infty$ with g and Δ fixed the total number of particles emitted is independent of T, thereby not giving rise to a golden rule behavior characterized by a linear growth in T. Second, $\langle N_\omega^V \rangle$ is maximum for $\omega \propto 1/\Delta$. Finally, for $\omega\Delta \gg 1$, one has $\langle N_\omega^V \rangle \simeq e^{-\pi\omega\Delta}$. We thus find all the expected attributes of transients: their particle content is independent of the duration T when $T/\Delta \gg 1$, and their Fourier content is peaked around the adiabatic switching rate Δ^{-1}.

We now study the space-time repartition of the energy fluxes associated with these transient effects. We first notice that once the D term defined in Eq. (75) has been dropped, the mean flux can be expressed as

$$\langle T_{VV} \rangle = -2\,\mathrm{Im}(\langle 0|T_{VV}L|0\rangle) + \mathrm{Re}(\langle 0|L[T_{VV},L]|0\rangle). \tag{91}$$

Of course, by decomposing L and T_{VV} in terms of creation and annihilation operators, one would recover, respectively, the linear and quadratic contributions of Eq. (81). However, being interested in the space-time properties, we shall not do so and shall work instead in the time "representation." In this approach, $\langle T_{VV} \rangle$ is governed by the V part of the (positive frequency) Wightman function. This latter obeys

$$\begin{aligned}\partial_V W(V-V') &= \partial_V \langle 0|\Phi^\dagger(V,U)\Phi(V',U')|0\rangle \\ &= -\frac{1}{4\pi}\frac{1}{V-V'-i\epsilon}.\end{aligned} \tag{92}$$

Using this function, the first term of Eq. (91) reads

$$\begin{aligned}\langle T_{VV} \rangle_{\mathrm{lin}} &= -8g\,\mathrm{Im}\left[\int dt\, f(t)\{\partial_t \partial_V W(V-t)\}^2\right] \\ &= \frac{g}{12\pi}\partial_t^3 f(t=V).\end{aligned} \tag{93}$$

To obtain this result, we have integrated by parts 3 times. The boundary contributions all vanish since f given in Eq. (88) decreases faster that any power of t. The last integration is trivially performed by using $\mathrm{Im}[(x-i\epsilon)^{-1}] = \pi\delta(x)$. These properties explain the local character of the expectation value.[4]

[4]It should be pointed out that we could have written $\langle T_{VV} \rangle_{\mathrm{lin}}$ as a commutator. This, however, is not appropriate since one loses the analytical properties of W which are encoded by $i\epsilon$ (they arise from frequency content of the vacuum and play a crucial role in defining the above expressions). By performing first the commutator [or, equivalently, by first taking the imaginary part in Eq. (93)], one would obtain an ill-defined expression. The same remark applies to the quadratic term in g. To obtain well-defined expressions, only one commutation (and not two) should be done.

To evaluate the second term of Eq. (91), which is quadratic in g, we proceed along the same lines. We first evaluate the commutator so as to obtain a quadratic form in Φ and Φ_V, where Φ_V means only that the V part of the field operator Φ should be kept. We then notice that the derivatives ∂_t in J might be expressed as ∂_V since they are evaluated at $z=0$, but they act both on the V and U parts of Φ. Using this notation, one gets

$$\begin{aligned}[T_{VV},L] &= igf(V)[(\partial_V \Phi_V^\dagger \partial_V^2 \Phi + \partial_V \Phi_V \partial_V^2 \Phi^\dagger) + \mathrm{H.c.}] \\ &\quad + igf'(V)[(\partial_V \Phi_V^\dagger \partial_V \Phi + \partial_V \Phi_V \Phi^\dagger) + \mathrm{H.c.}].\end{aligned} \tag{94}$$

Then the g^2 contribution of T_{VV} is

$$\begin{aligned}\langle T_{VV} \rangle_{\mathrm{quadr}} &= 16g^2 f(V)\mathrm{Re}\bigg(i\int dV' f(V')[\partial_{V'}\partial_V^2 W(V'-V)] \\ &\quad \times [\partial_{V'}\partial_V W(V'-V)]\bigg) + 16g^2 \partial_V f \\ &\quad \times \mathrm{Re}\bigg(i\int dV' f(V')[\partial_{V'}\partial_V W(V'-V)]^2\bigg) \\ &= -\frac{g^2}{12\pi}(f\partial_V^4 f + 2\partial_V f \partial_V^3 f).\end{aligned} \tag{95}$$

Having obtained explicit expressions for both terms of Eq. (91), we can now analyze the properties of $\langle T_{VV} \rangle$. First, neither f appears in Eq. (93) nor f^2 in Eq. (95). Thus one recovers the fact that an inertial mirror does not radiate while its coupling is constant. This is illustrated in Fig. 1. Second, being given by derivatives of f with respect to time, the magnitude of $\langle T_{VV} \rangle$ scales with positive powers of the switching on and off rate Δ^{-1}.

Finally, to obtain the integrated value of the energy emitted, as in Eq. (19), one decomposes $\langle T_{VV} \rangle$ into two parts: a total derivative which does not contribute to the total energy and the rest which turns out to be positive definite. Explicitly, we get

$$\begin{aligned}\langle T_{VV} \rangle &= \langle T_{VV} \rangle_{\mathrm{lin}} + \langle T_{VV} \rangle_{\mathrm{quadr}} \\ &= \frac{g^2}{12\pi}(\partial_V^2 f)^2 - \frac{1}{12\pi}\partial_V\bigg[-g\partial_V^2 f \\ &\quad + g^2\bigg(\frac{1}{2}\partial_V^4(f^2) - \partial_V^2[(\partial_V f)^2]\bigg)\bigg].\end{aligned} \tag{96}$$

Thus the total energy is

$$\langle H_V \rangle = \frac{g^2}{12\pi}\int_{-\infty}^{+\infty} dV (\partial_V^2 f)^2 = \int_0^\infty d\omega\, \omega \langle N_\omega^V \rangle. \tag{97}$$

Here $\langle H_V \rangle$ is finite when the mean number $\langle N_\omega \rangle$ decreases faster than ω^{-2}. This is the case when working with Eq. (88) at fixed $\Delta \neq 0$. In this case, one finds

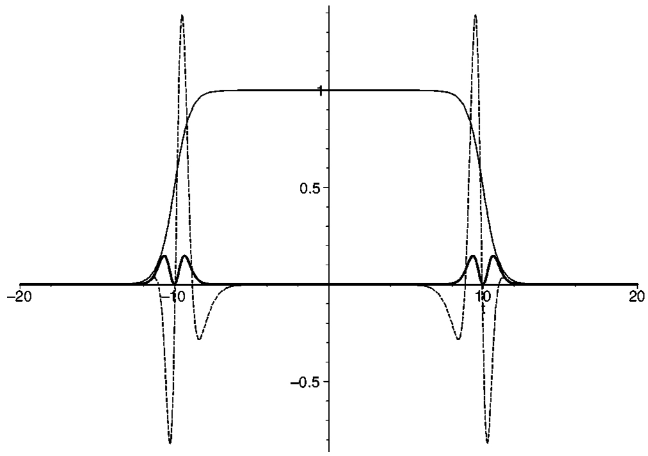

FIG. 1. The thin solid line is $f(t)$ given by Eq. (88) for $T=10$ and $\Delta=1$. The dashed line is $\langle T_{VV}\rangle_{\rm quadr}$, and the thick line is the part of $\langle T_{VV}\rangle$ which contributes to the energy; see Eq. (97). These two curves have been plotted in the same arbitrary units. The behavior of $\langle T_{VV}\rangle_{\rm lin}$ is similar to $\langle T_{VV}\rangle_{\rm quadr}$.

$$\langle H_V\rangle = \frac{2g^2}{45\pi}\frac{1}{\Delta^3}F\left(\frac{T}{\Delta}\right). \tag{98}$$

The main feature $\langle H_V\rangle$ is that it is independent of T in the limit $T/\Delta \gg 1$ (see Fig. 2), thereby confirming that the emitted energy is indeed associated with the two transitory periods, irrespectively of the lapse of time $(=2T)$ which separates them.

To conclude this subsection, we consider the limit $\Delta \to 0$. This corresponds to the situation studied in [21] and [7] in view of its analogies with the residual flux emitted at the end of the evaporation of a black hole. In this limit, $f(t)$ becomes a step function, the energy flux is concentrated in a narrow lapse Δ, and its frequency content diverges. In fact, $\langle T_{VV}\rangle$ becomes a *distribution* since it is built on the derivatives of $f(t)$. More precisely, the singularity is worse than a delta, as clearly seen from Eq. (95). This means that not only the instantaneous flux $\langle T_{VV}\rangle$ diverges, but also that the total energy emitted is singular, as indicated in Eq. (98).

Moreover, being singular, this singular behavior is not universal. It depends on the number of derivatives in the Hamiltonian, and it might also vary when considering higher orders in g. Hence the question as to what is the flux emitted by the disappearance of the reflection condition is not well defined. To have a well-defined question, one should first choose a regular model such as that defined by Eq. (54) and with $g(t)$ given by Eq. (88), and only then consider the singular limit $\Delta \to 0$. What we learn from this is that the DF model should be conceived as providing a useful approximative description of some physical processes only when the predictions are well defined, i.e., independent of the characteristics of the original dynamical model (such as the mass of the mirror or the precise nature of the coupling) when the limits of large mass and large coupling are taken.

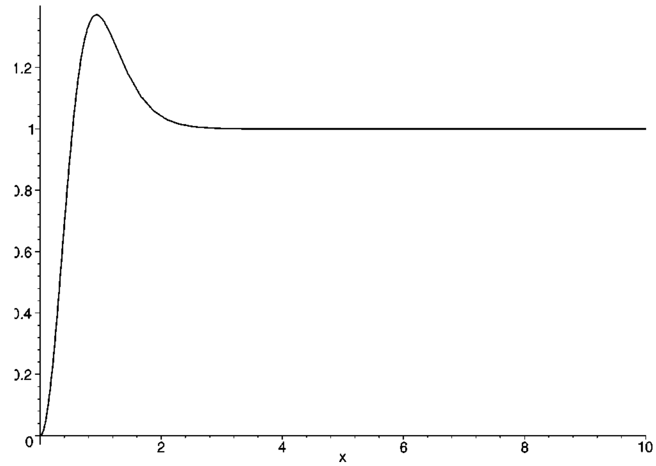

FIG. 2. The plot is $F(x)$ defined by Eqs. (98) and (97) in such a way that $F\to 1$ for $x\to\infty$.

B. Flux emitted by a uniformly accelerated mirror

The case of uniform acceleration in a Minkowski vacuum is *a priori* rather perplexing. On the one hand, Eq. (19) immediately gives that the mean flux vanishes for uniform acceleration, i.e., for $UV=-a^{-2}$. This is a consequence of the fact that ∂_τ is a Killing vector in Minkowski space-time [30–32]. On the other hand, however, the Bogoliubov coefficients $\beta_{\omega k}$ of Eq. (9) do not vanish [3,10]. Moreover, when used in Eq. (17), they lead to a divergent energy $\langle H_V\rangle$. To conciliate these results, one must infer that there is a singular flux on the past horizon $V=0$, as is the case for a uniformly accelerated "atom" coupled to the radiation field [33,8]. In fact, as shown in Appendix C of [17], this singular flux will be found for all uniformly accelerated quantum systems coupled to the radiation field.

Our aim is now to show that there is indeed a singular flux of energy along the past horizon when taking the limit of constant coupling $g(\tau)=g$ at the end of the calculation. To obtain the regularized expression for this flux, we shall generalize the analysis of the former section to a nonzero-temperature heat bath and then use the isomorphism between the flux emitted by this mirror at rest in a heat bath at temperature $a/2\pi$ and the flux emitted by a uniformly accelerated mirror of acceleration a when expressed in the Rindler coordinate system.

In a thermal bath, the V part of the Wightman function obeys

$$\partial_V W^\beta(V-V') = -\frac{1}{4\pi}\frac{\pi}{\beta}\coth\left(\frac{\pi}{\beta}(V-V'-i\epsilon)\right). \tag{99}$$

It reduces to $\partial_V W$ of Eq. (92) in the zero-temperature limit for $\beta\to\infty$. When replacing W by W^β in Eqs. (93) and (95), we obtain the mean flux emitted in a thermal bath. It can be shown to be[5]

[5]The details of the calculation will be presented in [23].

$$\langle T_{VV}\rangle^{\beta}=\frac{g}{12\pi}\partial_V^3 f-\frac{g^2}{12\pi}[f(V)\partial_V^4 f+2\partial_V f\partial_V^3 f]$$
$$-\left(\frac{2\pi}{\beta}\right)^2\left[\frac{g}{12\pi}\partial_V f-\frac{g^2}{12\pi}[f(V)\partial_V^2 f+2\partial_V f\partial_V f]\right]. \tag{100}$$

The first two terms are equal to Eq. (96), and the last two scale like $(\Delta/\beta)^2$. Thus they are negligible in the low-temperature limit $\beta \gg \Delta$ and dominant in the high-temperature regime.

We are now in position to obtain a regular expression for the flux emitted by a uniformly accelerated mirror in Minkowski vacuum. Using the well-known isomorphism between systems at rest in a thermal bath and accelerated systems in vacuum, the mean flux of Rindler energy emitted by a mirror of acceleration a is

$$\langle T_{vv}(v)\rangle^{\mathrm{acc}}=\langle T_{VV}(V=v)\rangle^{\beta=2\pi/a}, \tag{101}$$

where v is the null advanced Rindler time $[av=\ln(aV)]$ when the mirror is located in the right Rindler quadrant ($z>|t|$). When using Eq. (88), the coupling between the mirror and field is turned on during a proper-time lapse $2T$ and the switching on and off rate Δ^{-1} is now measured with the proper time τ.

In the limit $T\gg\beta$ and a^{-1}, $\langle T_{vv}(v)\rangle^{\mathrm{acc}}\to 0$ at fixed $|v|<T$ since the flux is localized in the transients of "thickness" Δ centered around $v=\pm T$. In this, we recover the fact that a uniformly accelerated mirror does not radiate. In the DF model, this immediately follows from Eq. (19). (As noted above, this vanishing is a universal property of accelerated systems when they have reached equilibrium with the Rindler bath [17].)

However, this vanishing flux is accompanied by transient effects whose Minkowski properties become singular in the limit $T\to\infty$ whatever the value of Δ is. This simply follows from the fact that the mean flux measured in the inertial system of coordinates V, $U=t\pm z$ is

$$\langle T_{VV}(V)\rangle^{\mathrm{acc}}=\left(\frac{dv}{dV}\right)^2\langle T_{vv}[v=a^{-1}\ln(aV)]\rangle^{\mathrm{acc}}$$
$$=\left(\frac{1}{aV}\right)^2\langle T_{vv}(v)\rangle^{\mathrm{acc}}. \tag{102}$$

From this expression, using Eq. (88), one finds that the Minkowski flux diverges for all T if $a>\Delta^{-1}$, i.e., if the switching on is slower than the boost factor $dv/dV=e^{-av}$, which diverges for $v\to-\infty$. When $a<\Delta^{-1}$, the flux is well defined and its maximal value, which grows like e^{2aT}, is reached around $V=a^{-1}e^{-aT}$. This establishes the fact that in the limit of constant coupling $T\to\infty$, one has a singular flux of energy along the past horizon. Quite surprisingly, the condition $a\Delta<1$ tells us that accelerated mirrors which lead to a finite Minkowski flux have their fluxes dominated by the first two terms in Eq. (100). That is, the flux is dominated by the boosted vacuum transients governed by Δ rather than by the temperature effects induced by the acceleration.

Notice that if one requires that the total Minkowski energy

$$\langle H_V^{\mathrm{acc}}\rangle=\int_{-\infty}^{+\infty}dv\; e^{av}\langle T_{vv}^{\mathrm{acc}}\rangle \tag{103}$$

and the mean number of Minkowski quanta emitted by the mirror be finite, one gets a weaker condition $a\Delta<2$. Indeed, only one power of the boost factor should be tamed by the switching-on function f. (A similar condition also arises when considering the fluxes emitted by an accelerated two-level atom [17].) When this condition is met, using Eqs. (100), (101), and (103), we get

$$\langle H_V^{\mathrm{acc}}\rangle=\frac{g^2}{12\pi}\int_{-\infty}^{+\infty}dv\; e^{av}[(\partial_v^2 f)^2+2a^2(\partial_v f)^2]. \tag{104}$$

This energy is positive definite and diverges, as expected, like e^{aT} when $aT\gg 1$.

V. CONCLUSIONS

In this paper we generalized the DF model which describes the scattering of a radiation field by a mirror which follows a noninertial trajectory. We first considered partially transmitting mirrors and then studied a dynamical model based on an action principle. We obtained the following results.

Equations (41) give the Bogoliubov coefficients in terms of the transfer matrix $\mathbf{T}_\lambda$ evaluated in the rest frame of the mirror and the overlaps $\alpha_{\omega\lambda},\beta_{\omega\lambda}$ which relate the Minkowski plane waves of frequency ω to the proper-time modes of (proper) frequency λ. This expression is *universal* in that it governs all quantum systems coupled linearly and stationarily to the radiation field. The only model-dependent quantity is $\mathbf{T}_\lambda$. This is illustrated by the dynamical model of Sec. III A, which gives rise to the diagonal transfer matrix given in Eq. (66).

The main difference between the partially transmitting mirrors defined in a purely kinematic way in Sec. III B and the dynamical model of Sec. III A concerns causality: see the discussion which follows Eq. (66).

In Sec. III B, we analyze the scattering in the interacting picture. In this picture, there is no Bogoliubov transformation since the basis of asymptotic states is provided by the usual "free" states engendered by the Minkowski creation operators. The nontrivial value of the energy flux emitted by the mirror results from the connected parts of the matrix elements of the evolution operator $\hat{S}=Te^{iL}$; see Eqs. (71)–(73).

Equations (52) and (53) as well as Eqs. (82) establish the connection between the Bogoliubov matrices α and β, which mathematically relate two bases of field modes, and the transition amplitudes for physical processes to occur, i.e., the matrix elements of $\hat{S}=Te^{iL}$. It should be noticed that a similar relationship also exists between Bogoliubov coefficients and the transition amplitudes of a two-level atom: see Eq. (2.55) in [8].

The usefulness of the interacting picture is that it permits us to switch on and off the coupling constant at some finite time in a controlled manner. This possibility in turn allows us to obtain regularized expressions for the flux in situations where the flux is ill defined when using the DF model. This is illustrated in Sec. IV B with the case of uniform acceleration.

APPENDIX: THE in-out OVERLAP IN THE GENERAL CASE

Our aim is to obtain an expression for the overlap between the in and out vacua when the Bogoliubov coefficients are nondiagonal. In this case, the original expression of Kamefuchi and Umezawa [34] does not apply.

In order to have simple expressions for this overlap, we will use a discretized basis of wavepackets in which the integrals are replaced by sums and Dirac distributions by Kronecker symbols. In addition to the in and out operators, it is appropriate to define a third class of operators $\hat{a}_\omega, \hat{b}_\omega$. This new basis generalizes the "Unruh" modes [27,8] in that $\hat{a}_\omega$ ($\hat{b}_\omega$) is made out of a_k^{in} (b_k^{in}), but is characterized by a fixed out frequency ω:

$$\alpha_\omega \hat{a}_\omega = \sum_k \alpha^*_{\omega k} a_k^{\mathrm{in}}, \quad \alpha_\omega \hat{b}_\omega = \sum_k \alpha^*_{\omega k} b_k^{\mathrm{in}}. \tag{A1}$$

The real coefficients α_ω are such that $[\hat{a}_\omega, \hat{a}^\dagger_\omega] = 1$; therefore, $\alpha_\omega^2 = \Sigma_k |\alpha_{\omega k}|^2$. The notion of particles and antiparticles is obviously maintained since the $\hat{a}$ are made of a^{in} only. Notwithstanding, for arbitrary $\alpha_{\omega k}$ and $\beta_{\omega k}$, this new basis is not orthogonal and the commutation rules are given by

$$[\hat{a}_\omega, \hat{a}^\dagger_{\omega'}] \equiv F_{\omega\omega'} = [\hat{b}_\omega, \hat{b}^\dagger_{\omega'}] = \frac{\Sigma_k \alpha^*_{\omega k} \alpha_{\omega' k}}{\alpha_\omega \alpha_{\omega'}}. \tag{A2}$$

By construction and from Eq. (13), these new operators are related to the out operators by

$$a_\omega^{\mathrm{out}} = \alpha_\omega \hat{a}_\omega - \sum_{\omega'} \alpha_{\omega'} B_{\omega\omega'} \hat{b}^\dagger_{\omega'},$$

$$b_\omega^{\mathrm{out}} = \alpha_\omega \hat{b}_\omega - \sum_{\omega'} \alpha_{\omega'} B_{\omega\omega'} \hat{a}^\dagger_{\omega'},$$

with

$$B_{\omega\omega'} \equiv \sum_k \beta_{\omega k} \alpha^{-1}_{k\omega'}, \tag{A3}$$

where $\alpha^{-1}_{k\omega}$ is the inverse matrix of $\alpha_{\omega k}$. ($\alpha_{\omega k}$ is always invertible since otherwise there would exist incoming particles whose scattering would give only antiparticles.)

As for the Unruh modes, the operators $\hat{a}$, $\hat{b}$ are useful to relate in a simple way the out vacuum to the in vacuum. Straightforward algebra indeed gives

$$|0_{\mathrm{out}}\rangle = \frac{1}{Z} \exp\left(\sum_{\omega\omega' k} \frac{\alpha_{\omega'}}{\alpha_k} F^{-1}_{\omega k} B_{k\omega'} \hat{a}^\dagger_\omega \hat{b}^\dagger_{\omega'} \right) |0_{\mathrm{in}}\rangle, \tag{A4}$$

where Z is defined by

$$Z^{-2} = |\langle 0_{\mathrm{out}} | 0_{\mathrm{in}} \rangle|^2. \tag{A5}$$

Even though Eq. (A4) looks cumbersome, one easily verifies that, to order β^2, it correctly gives the relationship between the vacuum decay ($Z > 1$) and the pair creation probability of Minkowski quanta. Indeed, using $(F^{-1})_{\omega\omega'} \langle 0_{\mathrm{in}} | \hat{a}_{\omega'} \hat{a}^\dagger_{\omega''} | 0_{\mathrm{in}} \rangle = \delta_{\omega,\omega''}$ and the condition on $B_{\omega\omega'}$ and $F_{\omega\omega'}$, which arises from $[a_\omega^{\mathrm{out}}, b_{\omega'}^{\mathrm{out}}] = 0$ and Eq. (A2), one obtains

$$Z^2 = 1 + \sum_{\omega\omega'} |B_{\omega\omega'}|^2 + O(\beta^4). \tag{A6}$$

This is the correct expression since the probability to have a pair of out quanta is

$$|\langle 0_{\mathrm{out}} | a_\omega^{\mathrm{out}} b_{\omega'}^{\mathrm{out}} | 0_{\mathrm{in}} \rangle|^2 = \left| \frac{B_{\omega\omega'}}{Z} \right|^2 = |B_{\omega\omega'}|^2 + O(\beta^4). \tag{A7}$$

For completeness, we notice that when the scattering is stationary (as is the case for uniform acceleration and in black hole evaporation), one has

$$B_{\omega\omega'} = \frac{\beta_\omega}{\alpha_\omega} \delta_{\omega,\omega'}, F_{\omega\omega'} = \delta_{\omega,\omega'}. \tag{A8}$$

Since they are diagonal, Eq. (A4) becomes

$$|0_{\mathrm{out}}\rangle = \frac{1}{Z} \exp\left(\sum_\omega \frac{\beta_\omega}{\alpha_\omega} \hat{a}^\dagger_\omega \hat{b}^\dagger_\omega \right) |0_{\mathrm{in}}\rangle, \tag{A9}$$

thereby recovering the usual diagonal expression governed by the "Unruh" operators $\hat{a}_\omega, \hat{b}_\omega$.

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